N. Dual-Bimodules and Nakayama Permutations

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A ring R is QF [1] if it is Artinian and admits a "Nakayama permutation" of its basic set $\{e_1, e_2, ..., e_n\}$ of primitive idempotents, i.e. a permutation σ on $\{1, 2, ... , n\}$ such that for each i, $1 \leq i \leq n$,

$$soc(Re_i) \cong Re_{\sigma(i)}/J(R)e_{\sigma(i)} \quad \text{and} \quad soc(e_{\sigma(i)}R) \cong e_i R/e_i J(R).$$

In [2] Azumaya has introduced the notion of quasi-Frobenius two-sided R-S-modules $_RQ_S$ and shown that if $_RR$ and S_S are both Artinian then these are characterized by the existence of a Nakayama permutation. Hajarnavis and Norton [3] have shown that every dual ring admits a Nakayama permutation and recently Nicholson and Yousif [4] also have shown that every minfull ring admits a Nakayama permutation.

In this note, unifying these results, we shall give in Section 1 a characterization of the Nakayama permutation using min injectiveity and then in Section 2 we shall characterize the dual-bimodules [5] using a Nakayama permutation and simple injectivity. In Section 3, using the factorization of σ as a product of disjoint cycles, we shall give a characterization of the dual-bimodules again.

The detailed version of this note will be submitted for publication elsewhere.

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