

Calculation of Characteristic Impedance for Stripline Based on Planar Circuit and Lateral Equivalent Network

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Abstract -- Stripline is a key component for MIC. Therefore, exact calculation of the characteristic impedance is important for analysis and synthesis of stripline circuit. There are many calculation methods for stripline with zero or finite strip thickness. Exact solution for zero thickness was already given by conformal transformation method, but that for finite thickness is not. So far, relatively exact solution is given by fringing capacitance method. Here, new and systematic mode analysis method is proposed, which is based on lateral equivalent network and can give more exact characteristic impedance for stripline with any thickness including zero thickness. This method is demonstrated to be valid and useful by good agreement with the exact results for zero thickness.

I. INTRODUCTION

Stripline is an important transmission line for MIC system. In order to carry out rigorous analysis and design for stripline circuit, exact characteristic impedance for any strip thickness is strongly needed. There are many numerical calculation methods for stripline with zero and finite strip thickness. Exact analytical solution for zero thickness was already well-known and given by conformal transformation method^[1], but that for finite thickness is not available. So far, relatively exact solution are given by fringing capacitance method^[2], which has 1.2% error. Hence, it is important to develop a new method which can give more exact characteristic impedance for any strip thickness including zero. In this paper, a new and systematic method for the calculation of propagation eigenmode (propagation constant and field distribution of TEM mode) for stripline based on lateral equivalent network^[4] will be proposed, which is derived from planar circuit equations^[3] and mode matching method, and can give exact field distribution for TEM mode. Then characteristic impedance is defined by TEM voltage and current, which can be calculated by line integral of the electric field and contour integral of magnetic field, respectively. These new calculation method is applied to actual stripline structure, dispersion characteristics, field distribution and characteristic impedance are calculated for various dimension. These calculated results are compared with exact solution of zero thickness or conventional solution with good agreement, which demonstrates that our new method is valid and usefull.

II. LATERAL EQUIVALENT NETWORK^[4]

Cross-section of stripline is divided into three uniform regions and two step discontinuities as shown in Fig.1(a). In order to analyze the stripline with any strip thickness, equivalent network in lateral direction is introduced, which is derived from parallel plate planar circuit equations^[3] and dependency of $e^{-j\beta_n y}$ along waveguide. Parameters of the planar circuit equations are given in table 1. The equivalent network for uniform region is given by multi-transmission line of TE/TM mode and that for step discontinuity by multi-port ideal transformer with mode conversion current source at both side of the step as shown in Fig.1(b)^[4]. Therefore whole equivalent network in lateral direction is given by Fig.1(c). Through our calculation, half structure of the stripline is treated because of the symmetry at center. Also metal wall is assumed at outside side-wall to make the model simple.

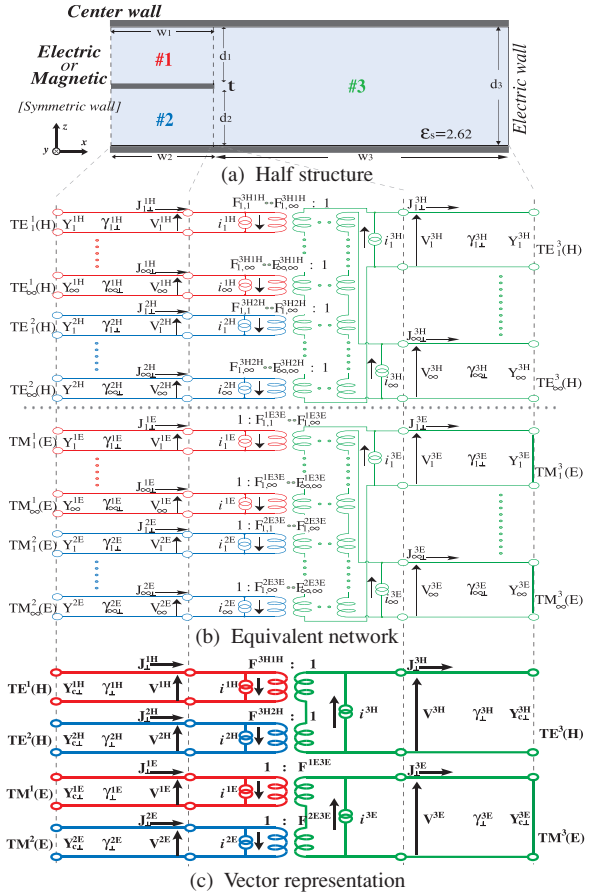


Fig.1 Lateral equivalent network for stripline

III. CALCULATION OF PROPAGATION EIGENMODE

For the calculation of propagation eigenmode, eigen-value equation given by eq.(1) is formulated by mode matching condition and lateral equivalent network given by Fig.1.

$$\begin{bmatrix} \bar{\mathbf{Y}}_{eff}^{1E1E} + \bar{\mathbf{Y}}_{in}^{1E,2} & \bar{\mathbf{Y}}_{eff}^{1E2E} & \bar{\mathbf{Y}}_{eff}^{1E3H} \\ \bar{\mathbf{Y}}_{eff}^{2E1E} & \bar{\mathbf{Y}}_{eff}^{2E2E} + \bar{\mathbf{Y}}_{in}^{2E,2} & \bar{\mathbf{Y}}_{eff}^{2E3H} \\ \bar{\mathbf{Y}}_{eff}^{3H1E} & \bar{\mathbf{Y}}_{eff}^{3H2E} & \bar{\mathbf{Y}}_{eff}^{3H3H} + \bar{\mathbf{Y}}_{in}^{3H,1} \end{bmatrix} \begin{bmatrix} \mathbf{V}^{1E,2} \\ \mathbf{V}^{2E,2} \\ \mathbf{V}^{3H,1} \end{bmatrix} = 0 \quad \dots (1)$$

Propagation constant can be obtained by solving eq.(2).

$$\det(\mathbf{Y}_{eff} + \mathbf{Y}_{in}) = 0 \quad \dots (2)$$

Field distribution of the eigenmode is calculated by eigenvector given by eq.(1). That is, all terminal mode voltage/current are obtained by the eigen-vector and whole equivalent network in lateral direction. Then, any x dependent mode voltage and mode current distribution is given by eq.(3), where $V_p^{1,2}$ and $I_p^{1,2}$ are terminal voltage/current.

$$\begin{aligned} V_p(x) &= \frac{\sinh \gamma_p(x - \ell_2)}{\sinh \gamma_p(\ell_1 - \ell_2)} V_p^{1,2} + \frac{\sinh \gamma_p(x - \ell_1)}{\sinh \gamma_p(\ell_2 - \ell_1)} V_p^{2,2} \\ \bar{I}_p(x) &= \frac{\sinh \gamma_p(x - \ell_2)}{\sinh \gamma_p(\ell_1 - \ell_2)} \bar{I}_p^{1,2} + \frac{\sinh \gamma_p(x - \ell_1)}{\sinh \gamma_p(\ell_2 - \ell_1)} \bar{I}_p^{2,2} \quad \dots (3) \end{aligned}$$

Hence, field distribution of the corresponding eigenmode at each uniform region is given by TE/TM mode summation of product of z-dependent mode function in height direction and x dependent mode voltage and mode current distribution. Final field distribution is given by eqs.(4) ~ (9).

$$E_z(x, z) = -\frac{1}{d} \sum_n V_n^E(x) \cdot g_n^E(z) \quad \dots(4)$$

$$H_z(x, z) = -\frac{1}{d} \sum_m V_m^H(x) \cdot g_m^H(z) \quad \dots(5)$$

$$E_y(x, z) = -\sum_m J_{y,m}^H(x) \cdot f_m^H(z) + j\eta_0 \sum_n J_{y,n}^E(x) \cdot h_n^E(z) \quad \dots(6)$$

$$H_y(x, z) = \sum_n J_{x,n}^E(x) \cdot f_n^E(z) + j \frac{1}{\eta_0} \sum_m J_{x,m}^H(x) \cdot h_m^H(z) \quad \dots(7)$$

$$E_x(x, z) = \sum_m J_{y,m}^H(x) \cdot f_m^H(z) + j\eta_0 \sum_n J_{x,n}^E(x) \cdot h_n^E(z) \quad \dots(8)$$

$$H_x(x, z) = -\sum_n J_{y,n}^E(x) \cdot f_n^E(z) + j \frac{1}{\eta_0} \sum_m J_{x,m}^H(x) \cdot h_m^H(z) \quad \dots(9)$$

IV. CALCULATION OF CHARACTERISTIC IMPEDANCE

TEM mode characteristic impedance of stripline, whose structure is shown in Fig.2 is defined by eq.(10), where TEM voltage V^{TEM} and current I^{TEM} is derived by line integral of electric field E_z given by eq.(11) and contour integral of magnetic field H_t given by eq.(12), respectively.

$$Z_c = \frac{V^{TEM}}{I^{TEM}} \quad \dots(10)$$

$$V^{TEM} = \int_0^A E_z dz \quad \dots(11)$$

$$I^{TEM} = 2 \int_{ABCD} \mathbf{H}_t \cdot ds \quad \dots(12)$$

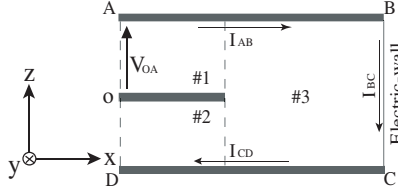


Fig.2 Cross section of half stripline

Exact electric field E_t and magnetic field H_t in the cross-section of stripline can be calculated by solving eigenvalue equation and substituting eigenvector into eq.(4) through eq.(9) as explained in the previous section. Definite integral method by terminal mode voltage and mode current based on lateral equivalent network as shown in Fig.3 is proposed and explained in the following.

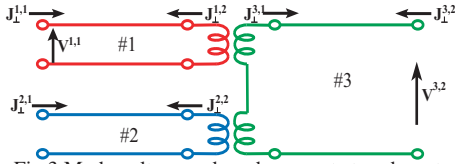


Fig.3 Mode voltage and mode current at each port

TABLE 1 Relations and parameter for planar circuit

	z-dependent function	x-y dependent function
TM(E) mode	$f_n^E(z) = \varepsilon_n g_n^E(z) = \sqrt{\varepsilon_n} \sqrt{\varepsilon_n} \cos\left(\frac{n\pi}{d} z\right)$	$V_n^E(x, y) = -E_{zn}^H(x, y) \cdot d[V]$
	$h_n^E(z) = \left(\frac{n\pi}{k_0 d}\right) \frac{1}{\sqrt{\varepsilon_n}} \sqrt{\varepsilon_n} \sin\left(\frac{n\pi}{d} z\right)$	$\mathbf{J}_n^E(x, y) = \mathbf{H}_n^H(x, y) \times \mathbf{k}[A/m]$
	$\gamma_m^E = \sqrt{\left(\frac{n\pi}{d}\right)^2 - \varepsilon_n k_0^2} = j\beta_m^E$	$Y_{cn}^E = \sqrt{\frac{B_n^E}{X_n^E}} = \frac{\omega \varepsilon_0}{\beta_m^E} \frac{1}{d}[S/m]$
	$\varepsilon_n = \begin{cases} 1 & (n=0) \\ 2 & (n \neq 0) \end{cases}$	$\beta_m^E = \sqrt{X_m^E B_m^E}[rad/m]$
TE(H) mode	$f_m^H(z) = g_m^H(z) = \sqrt{2} \sin\left(\frac{m\pi}{d} z\right)$	$V_m^H(x, y) = H_{zm}^H(x, y) \cdot d[A]$
	$h_m^H(z) = \left(\frac{m\pi}{k_0 d}\right) \sqrt{2} \cos\left(\frac{m\pi}{d} z\right)$	$\mathbf{J}_m^H(x, y) = \mathbf{k} \times \mathbf{H}_m^H(x, y)[V/m]$
	$\gamma_{mm}^H = \sqrt{\left(\frac{m\pi}{d}\right)^2 - \varepsilon_s k_0^2} = j\beta_{mm}^H$	$Y_{cm}^H = \sqrt{\frac{B_m^H}{X_m^H}} = \frac{\omega \mu_0}{\beta_{mm}^H} \frac{1}{d}[\Omega/m]$
		$\beta_{mm}^H = \sqrt{X_m^H B_m^H}[rad/m]$
		$X_m^H = (\beta_{mm}^H)^2 \cdot d / \omega \mu_0[\Omega]$
		$B_m^H = \omega \mu_0 / d[S/m^2]$

Transverse field E_t (E_x, E_z) and H_t (H_x, H_z) are given analytically by eqs.(4),(5),(8) and (9), which make possible to use definite integral for eqs.(11) and (12). Therefore, TEM voltage V^{TEM} is calculated analytically by the following definite integral.

$$V_{OA}^{TEM} = -\frac{1}{d} \int_0^A \sum_n V_n^E(x) \cdot g_n^E(z) dz$$

$$= -\frac{1}{d} \int_0^A \sum_n \frac{\sinh \gamma_p(x - \ell_2)}{\sinh \gamma_p(\ell_1 - \ell_2)} V_p^1 \cdot g_n^E(z) dz \quad \dots(13)$$

Also TEM current given by eq.(12) is calculated analytically by the following definite integral.

$$I_{ABCD}^{TEM} = 2(I_{AB}^{TEM} + I_{BC}^{TEM} + I_{CD}^{TEM}) \quad \dots(14)$$

$$I_{AB}^{TEM} = \int_A^B \left[-\sum_n J_{y,n}^E(x) \cdot f_n^E(z) + j \frac{1}{\eta_0} \sum_m J_{x,m}^H(x) \cdot h_m^H(z) \right] ds \quad \dots(15)$$

$$I_{BC}^{TEM} = -\frac{1}{d} \int_B^C \sum_m V_m^H(x) \cdot g_m^H(z) ds \quad \dots(16)$$

$$I_{CD}^{TEM} = \int_C^D \left[-\sum_n J_{y,n}^E(x) \cdot f_n^E(z) + j \frac{1}{\eta_0} \sum_m J_{x,m}^H(x) \cdot h_m^H(z) \right] ds \quad \dots(17)$$

The accuracy depends only on number of slab-mode considered.

V. RESULTS OF CALCULATION

So far explained analysis method is applied for practical strip line of various dimension and frequency to prove the validity. A. Convergence behavior of propagation constant

The keypoint of our method is how many height-mode must be taken into consideration, which is decided by convergence behavior of propagation constant with height-mode. The convergence behavior of TEM and two TE lower modes with height-mode in region #3 are shown in Fig.4 for 0.1[mm].

B. Calculation of Dispersion Characteristics

Eigenvalue equation given by eq.(1) is solved to give the eigenvalue and eigen-vector for various dimension including zero thickness. Calculated results of lower eigenvalue for zero thickness, which corresponds to propagation constant are shown in Fig. 5 as a function of frequency up to 60[GHz]. The calculated propagation constant for TEM mode agrees with exact result of $\beta = \omega \sqrt{\varepsilon \mu}$. Through our calculation 70 TE/TM slab-modes are taken into consideration.

C. Field Distribution for TEM Mode

Field distribution for TEM mode is calculated based on eigen vector, lateral equivalent network and eqs.(4)~(9). The calculated results of transverse field distribution (x and z field component) for various dimension are shown in Fig.6. Electric and magnetic field strength(E_y/H_y) of this mode in y direction is calculated to be about 10^{-13} , which means the mode is TEM.

The electric and magnetic field E_x, E_y, E_z and H_x, H_y, H_z at #1-#3 and #2-#3 step must be continuous because of field theory. Continuity of the H_x field for 0.1[mm] is calculated and shown in Fig.7 with 10 and 80 height-modes in region #3 considered.

D. Investigation of Error for Characteristic Impedance

In order to investigate the accuracy of our method, characteristic impedance of TEM mode for zero thickness are calculated by our method with definite integral and shown in Fig.8 with exact results given by conformal transformation^[1] and static fringing capacitance approximation^[2]. It turns out that our present method agree well with exact solution rather than popular static fringing capacitance approximation.

In order to investigate the calculation error in detail, calculation % error for each method is calculated and shown in Fig.9, which clearly demonstrates that the present method with definite

integral is better method giving maximum error of 0.17%. Finally characteristic impedance vs w/d relation are calculated and shown in Fig.10 as a function of w/d with parameter of thickness. It can be seen that the effect of thickness on the characteristic impedance is sensitive. Characteristic impedance for very thin strip (a few percent of the plate spacing) is calculable by this method and shown in the same figure as a function of w/d .

VI. CONCLUSION

New and systematic analysis method for stripline is presented, which is based on planar circuit and lateral equivalent network, and can give exact eigenmode for the stripline. How to calculate characteristic impedance of TEM mode is proposed and confirmed with success by applying the proposal to the practical problem. Also calculation error by this method is investigated by comparison with exact value given for $t=0$ [mm]. The agree-

ment is good, which demonstrates the validity of this method. Also this method can give more exact characteristic impedance compared with that given by conventional fringing capacitance method whose accuracy is 1.2%. Thus obtained more exact characteristic impedance is useful for exact analysis and design of stripline circuit.

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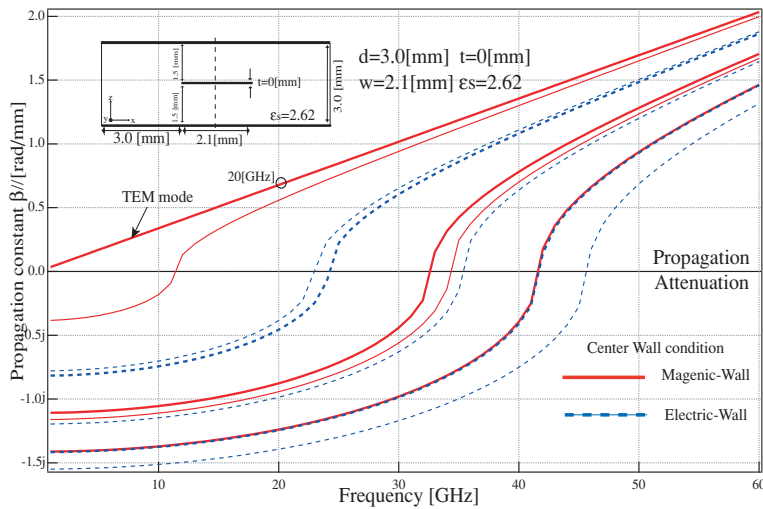


Fig.5 Frequency characteristics of propagation constants of TEM mode and higher modes for stripline of zero thickness($t=0.1$ [mm]) where 70 TE/TM slab-modes are taken into consideration.

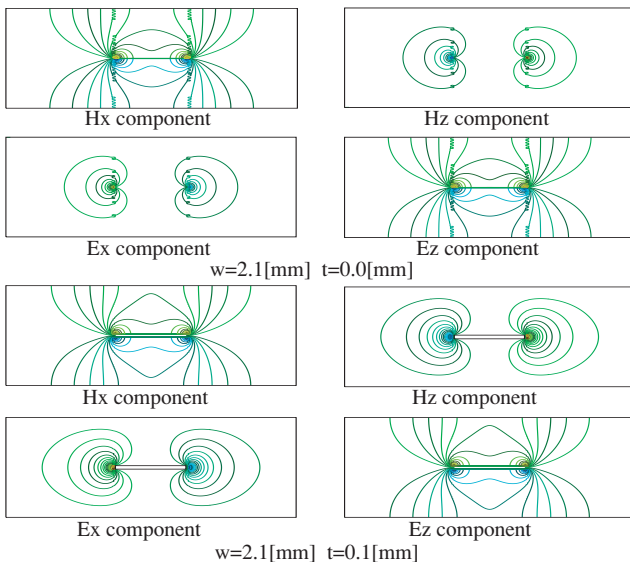


Fig.6 Distribution of transverse electromagnetic field E_t (E_x and E_z) and H_t (H_x and H_z) for various dimension of the strip where the calculation is carried out at 20 [GHz] and 70 TE/TM slab-modes are taken into consideration.

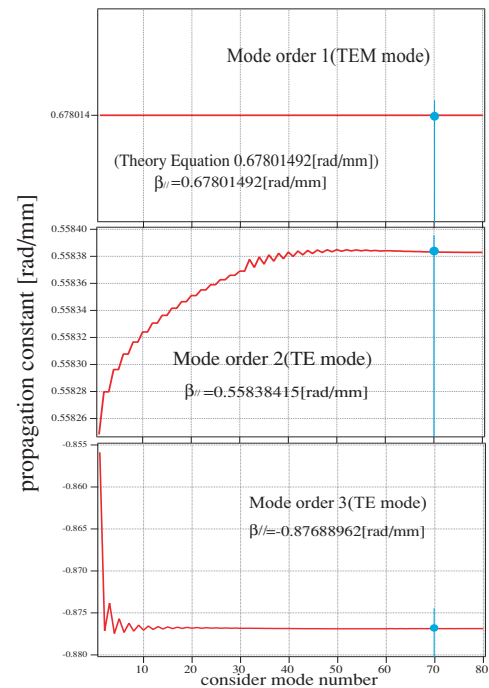


Fig.4 Convergence of propagation constant for stripline of 0.1[mm] thickness with number of height mode in #3 region

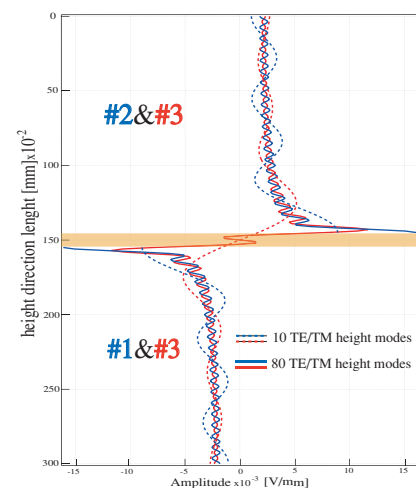


Fig.7 Continuity of the field component H_x at step discontinuities #2-#3 and #1-#3 for 0.1[mm] thickness strip.

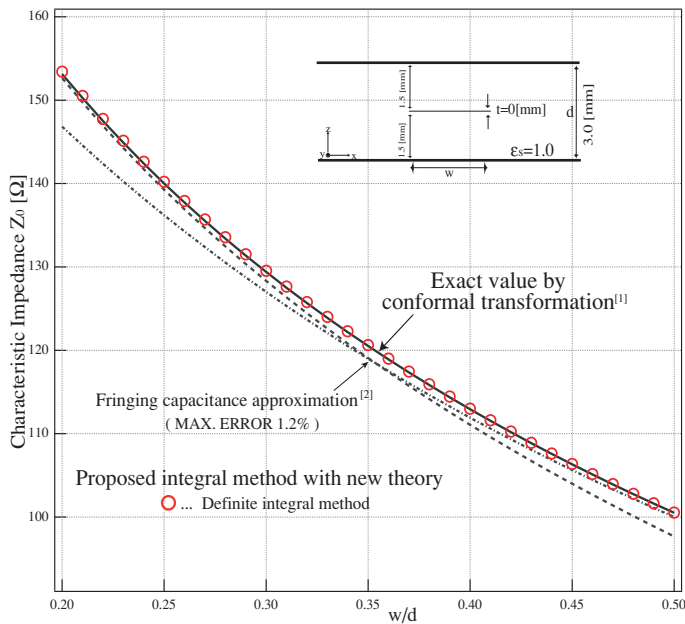


Fig. 8 Characteristic impedance vs w/d characteristics based on new theory/fringing capacitance approximation/exact conformal transformation for zero strip thickness

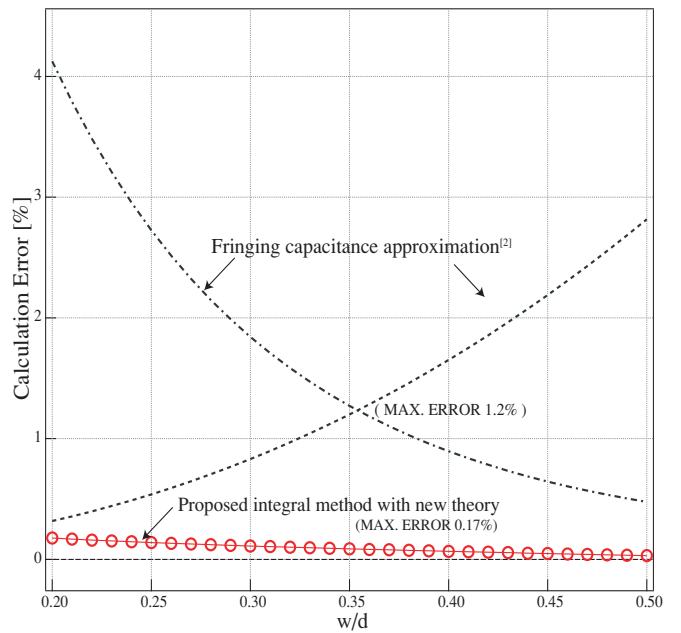


Fig. 9 Calculation error in % vs w/d characteristics for new theory and fringing capacitance approximation for zero strip thickness based on exact conformal transformation

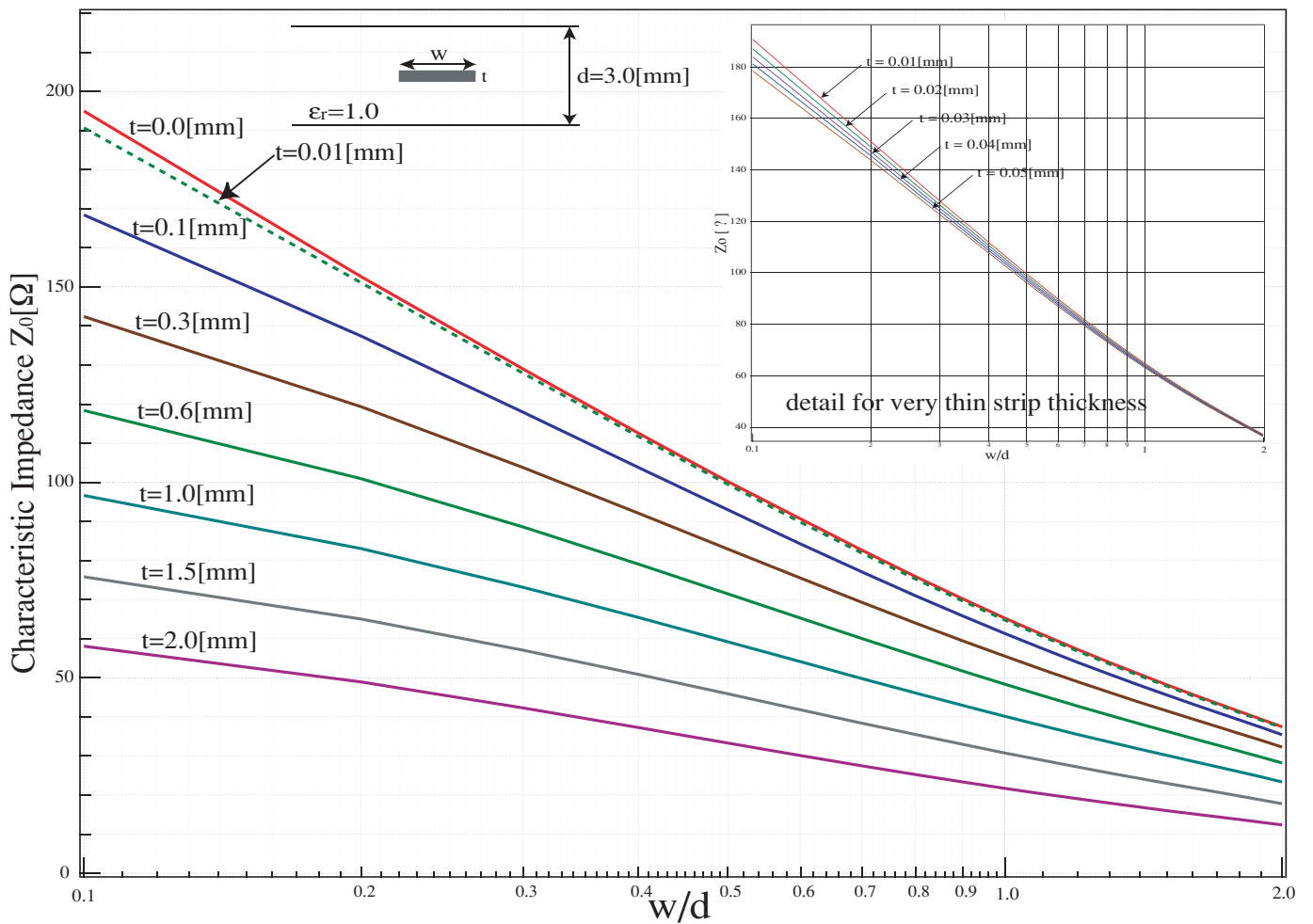


Fig.10 Characteristic impedance Z_0 vs w/d characteristics with parameter of strip thickness t and their detailed characteristics for very thin strip thickness t