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Analysis of stripline T-junction with rectangular cut based on eigenmode expansion method and Foster-type equivalent network

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Abstract — **Stripline T-junction with rectangular cut at symmetric center is analyzed based on two dimensional (2-D) planar circuit model, mode theory and thus derived Foster-type equivalent network. Wide-band frequency characteristics are calculated with cut-size as a parameter, which leads to discussion about transmission vs bandwidth characteristics, realized limit of characteristics and determination of the optimum cut.**

I. Introduction

Stripline T-junction with rectangular cut at symmetric center as shown in Fig.1(a) is analyzed by mode theory and Foster-type equivalent network according to the following steps.

1. Stripline T-junction can be modelled to 2-D planar circuit with magnetic side wall.
2. 2-D planar circuit is divided into planar waveguide section and planar junction section.
3. Applying eigenmode expansion method for each section, equivalent multi-transmission line network is derived for waveguide section and Foster-type equivalent network for junction section, respectively.
4. Planar junction section can be modelled to two planar waveguide connected with step discontinuity.

Wide-band frequency characteristic are calculated from Foster-type equivalent network with cut-size C as a parameter, and the optimum performance is discussed in terms of transmission vs bandwidth relation. Finally, the optimum cut is estimated roughly to be 0.6, but it is pointed out that there exists some kind of competetion between transmission and band-width .

II. Derivation of equivalent network for stripline discontinuity based on mode theory

Stripline discontinuity can be modelled by planar circuit, which can be divided into planar waveguide section (ℓ^i-s^i local coordinate system, i=1,2,3) and planar junction section (x-y coordinate system) as shown in Fig.1(a). Equivalent network for each section is derived based on mode theory.

(1) *Equivalent network for planar waveguide*

When width mode function for i-th planar waveguide in Fig.1(a) is defined by eq.(1), p-th mode voltage V_p^i and mode current I_p^i can be properly defined, and related by conventional transmission line equations.

$$f_p^i(s^i) = \sqrt{\varepsilon_p} \cos\frac{p\pi}{W^i} s^i \qquad (1)$$

$$\varepsilon_p = 1 \quad (p=0), \quad 2 \quad (p \geq 1) \qquad (p = 0,1,2,\cdots)$$

Therefore, the equivalent multi-transmission line network is shown in Fig.1(b), where mode propagation constant γ_p^i and characteristic mode impedance ${Z_C}_p^i$ are given by following equations.

$$\gamma_p^i = \sqrt{(p\pi/W^i) - \omega^2\varepsilon\mu}\ , \quad {Z_C}_p^i = \frac{j\omega\mu}{\gamma_p^i}\frac{d}{W^i} \quad [\Omega] \qquad (2)$$

In order to represent multi-transmission line simply, vector notation in bold line is introduced as shown in Fig.1(c) ,where $\boldsymbol{v}^i$ and $\boldsymbol{i}^i$ are mode voltage and current column matrix, γ^i and $\boldsymbol{Z}_C^i$ are propagation constant and characteristics mode impedance in diagonal matrix.

(2) *Equivalent network for planar junction*

When eigenmode function defined by Table 1 are once calculated for planar junction S, Foster-type equivalent network shown in Fig.1(b) is derived by mode expansion method, whose vector notation is also given by bold line in Fig.1(c). Network parameters of n-th mode capacitance, inductance and ideal transformer ratio between n-th mode in the planar junction and p-th mode in i-th waveguide are given by eqs.(3).

$$C_0 = \varepsilon\frac{S}{d} \quad [\mathrm{F}] \qquad L_n = \frac{\varepsilon\mu}{k_n^2}\frac{1}{C_0} \quad [\mathrm{H}]$$

$$n_{n\,p}^{\,i} = \frac{1}{W_i}\int_0^{W_i} \phi_n(x,y) f_p^i(s^i)\, ds^i \qquad (3)$$

Hence, the whole equivalent network for 3-port T-junction circuit is given by Fig.1(c) in vector notation.

III. Calculation of eigenmode for rectangular shaped planar circuit with rectangular cut

Rectangular-shaped planar junction with rectangular cut as shown in Fig.2(a) has symmetric structure, and

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so eigenmodes can be given by calculation for half structure with magnetic or electric wall at symmetric center. Half structure can be understood as two planar waveguides connected with step. Planar waveguides and coupling between width mode of two waveguides at step can be represented by multi-transmission line and multiport ideal transformer as shown in Fig.2(*b*). Network parameters for the former are given by eq.(2) and that for latter by eq.(4),where $f_n^1(x)$, $f_m^2(x)$ are width mode function given by eq.(1).

$$n_{nm}^{21} = \frac{1}{W_s}\int_0^{W_s} f_n^2(x) f_m^1(x)\, dx \qquad (4)$$

Hence, whole equivalent network in vector notation is given in Fig.2 (*c*). Mode matching technique is applied for calculation of eigenmode. Input mode admittance matrix looking toward left side and right side at port 2 of waveguide #1 $\overleftarrow{Y}_{in}, \overrightarrow{Y}_{in}$ are easily given by eqs.(6) and (7) after network analysis, and then the mode matching equation is given by eq.(5), where $\boldsymbol{v}^{1,2}$ is mode voltage column matrix at port 2 of waveguide #1.

$$\left(\overleftarrow{Y}_{in} + \overrightarrow{Y}_{in}\right) v^{1,2} = 0 \quad (5) \qquad \overleftarrow{Y}_{in} = Y_c^1 \coth \gamma^1 \ell^1 \quad (6)$$

$$\overrightarrow{Y}_{in} = n^{1,2}\left\{Y_c^2 \tanh \gamma^2 \ell^2 (open) \text{ or } Y_c^2 \coth \gamma^2 \ell^2 (short)\right\}\left(n^{1,2}\right)^t \quad (7)$$

Eigenvalue and eigenfunction are given by solving this eigenvalue equation (5). After investigating the convergence behavior of eigenvalue with number of width mode in waveguides, 30 modes is determined to be taken into consideration for wider waveguide. The eigenvalues up to *ka*=30 are calculated and shown in Fig.4 as a function of cut parameter C(=*c*/*a*). Also using the eigenvector for eq. (5) and equivalent network shown in Fig.2(*b*), mode voltages and currents at each port of planar waveguide can be calculated , which leads to the calculation of mode function. That is, combining with the corresponding width mode function, mode field distribution can be calculated. As example 1st mode for various cut parameters are shown in Fig.3.

Table.1 Eigenmode system

$$\frac{\partial \phi_n}{\partial x^2} + \frac{\partial \phi_n}{\partial y^2} + k_n^2 \phi_n = 0 \quad \text{in } S$$

$$\boldsymbol{n} \cdot grad\phi_n = 0 \quad \text{on } C \quad (\text{entire circumference of } S)$$

$$k_0 = 0, \ k_1 \le k_2 \le \cdots \quad n = 0, 1, 2, \cdots$$

$$\frac{1}{S}\iint_S \phi_n \cdot \phi_m dxdy = \delta_{nm} \qquad (\text{orthonomal})$$

where S is the area of the planar circuit

IV. Calculation of the frequency characteristics - determination of the optimum cut -

Based on the above mode calculation, network parameters in Fig.1(b) are calculated from eqs.(2),(3). Then, the frequency characteristics of the T-junction circuit with rectangular cut are calculated based on the conventional network theory. The essential problem in these calculations is how many modes in the waveguide and junction must be taken into consideration (truncation error). After investigating the convergence behavior with number of mode in both regions, it is determined to take up-to 5th waveguide mode and up-to *ka*=10 planar junction mode (17 modes for 0.60 cut parameter). Final calculated results of the frequency characteristics are shown in Fig.5 where 10 width modes in narrow waveguide and up-to *ka*=10 planar junction modes are taken into consideration. From this figure it turns out that the optimum cut parameter is about 0.60. The calculation in more detail about 0.60 is given also in Fig.6, which shows that around cut parameter of 0.60 the larger the cut parameter, the narrower the bandwidth but better transmission.

V. Discussion and conclusion

By general and systematic calculation method, wideband frequency characteristics of T-junction with rectangular cut are calculated. From the calculated results we can see that the optimum cut is about 0.6 and the junction works like lowpass filter about 0.6. We will try to determine the optimum shape and dimension for better T-junction in future, because present software can be used for any dimension.

Reference

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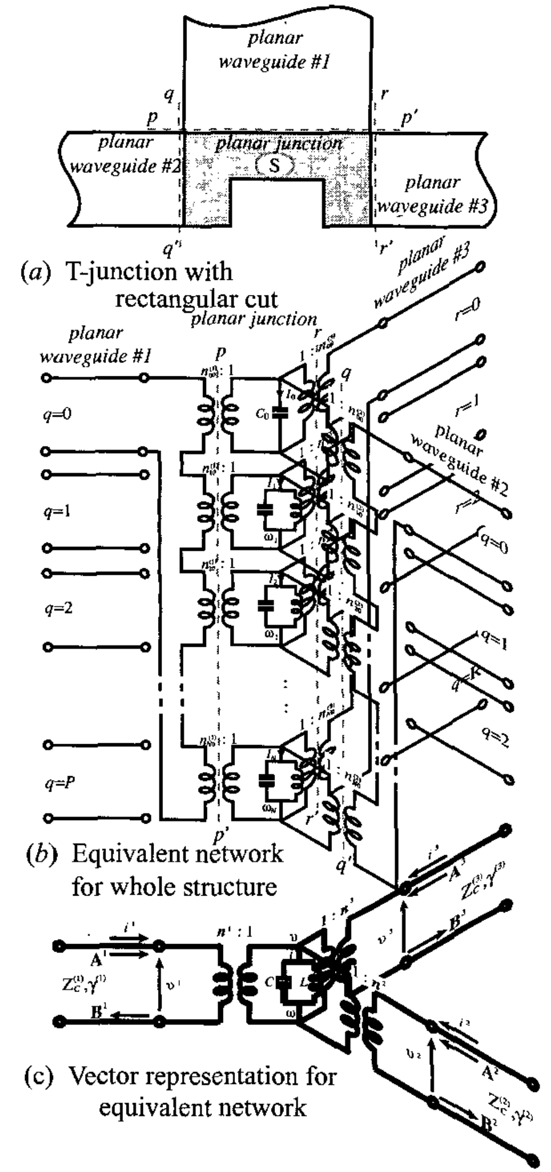
[2] Hsu, Jui-Pang, T. Hiraoka, Y. Tabei "Analysis of Stripline Right-angle Bend with slant-wise corner cut based on eigenmode expansion method and Foster-type equivalent network" 1999 IEEE MTT-S Digest WEF2-2

[3] Hsu, Jui-Pang "Analysis method for electromagnetic wave problems edited by E.Yamashita" chap.6, pp226-251 Artech house

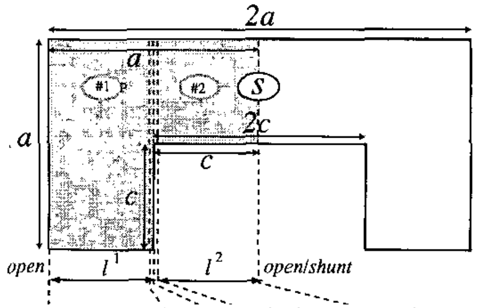


(*a*) T-junction with rectangular cut

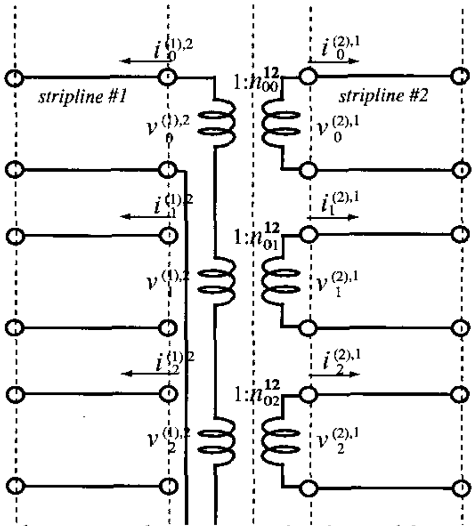
(*b*) Equivalent network for whole structure

(c) Vector representation for equivalent network

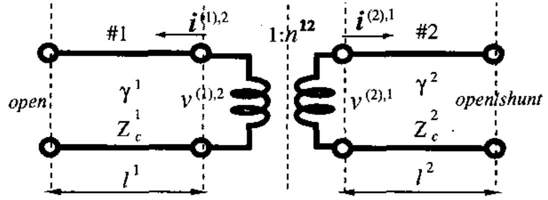
Fig.1 Planar circuit model for T-junction with rectangular cut and its equivalent network representaion



(a) rectangular-shaped planar junction with rectangular cut

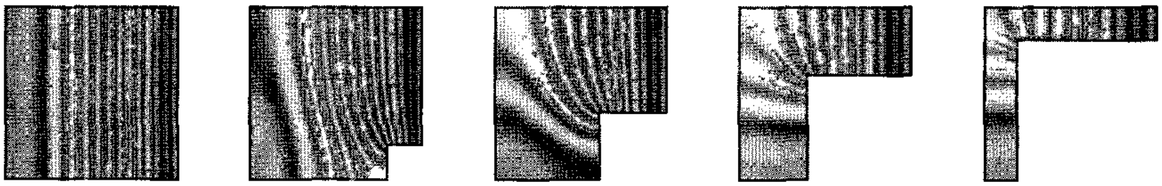


(*b*) Equivalent network representation by multi-transmission lines and multiport ideal transformers



(*c*) Vector representation for equivalent network

Fig.2 Equivalent network representation for rectangular shaped planar circuit with rectangular cut



[1]C=0.01 [2]C=0.2 [3]C=0.4 [4]C=0.6 [5]C=0.8

Fig.3 Field distribution of first eigenmode for various cut parameters

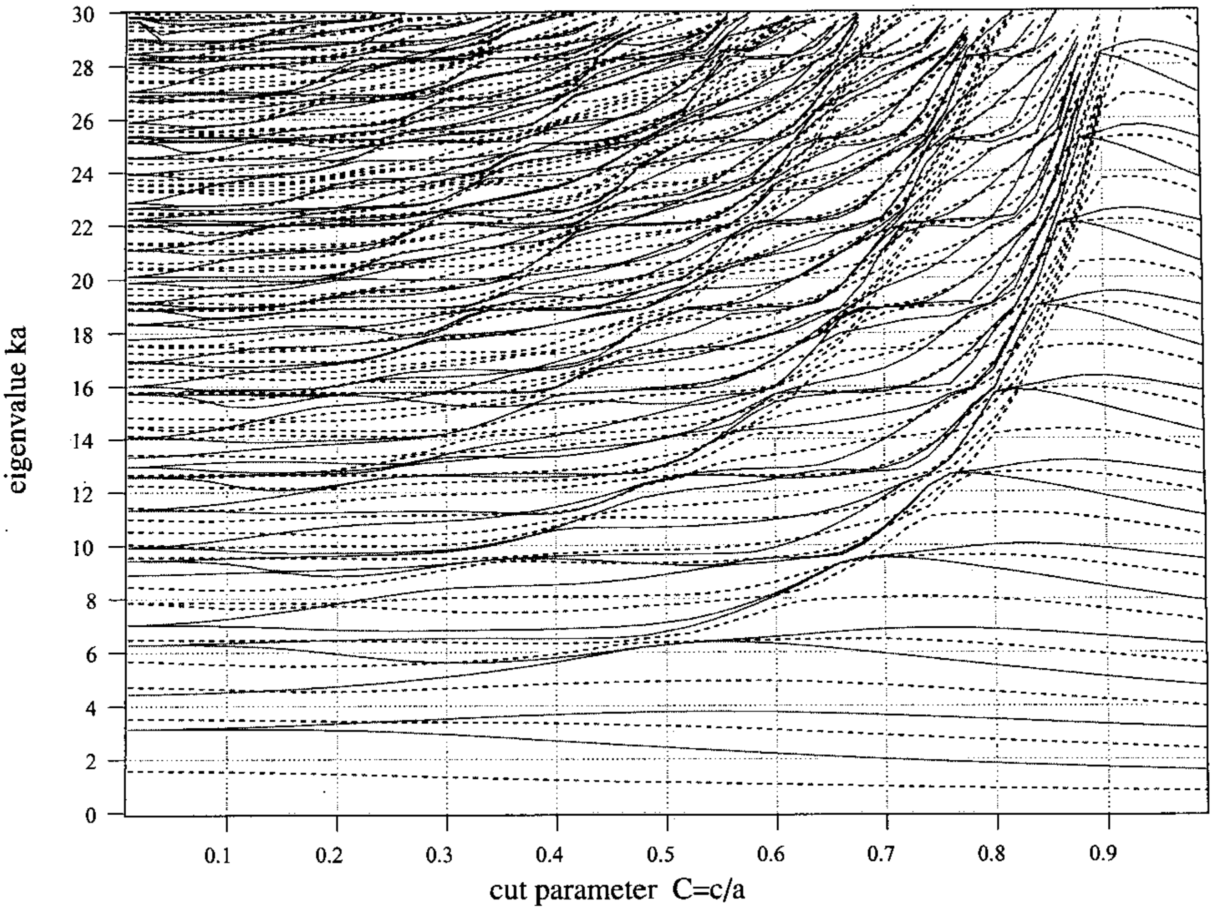


Fig.4 Eigenvalue for rectangular shaped planar circuit with rectangular cut
(Solid line is magnetic wall and broken line is electric wall at the center)

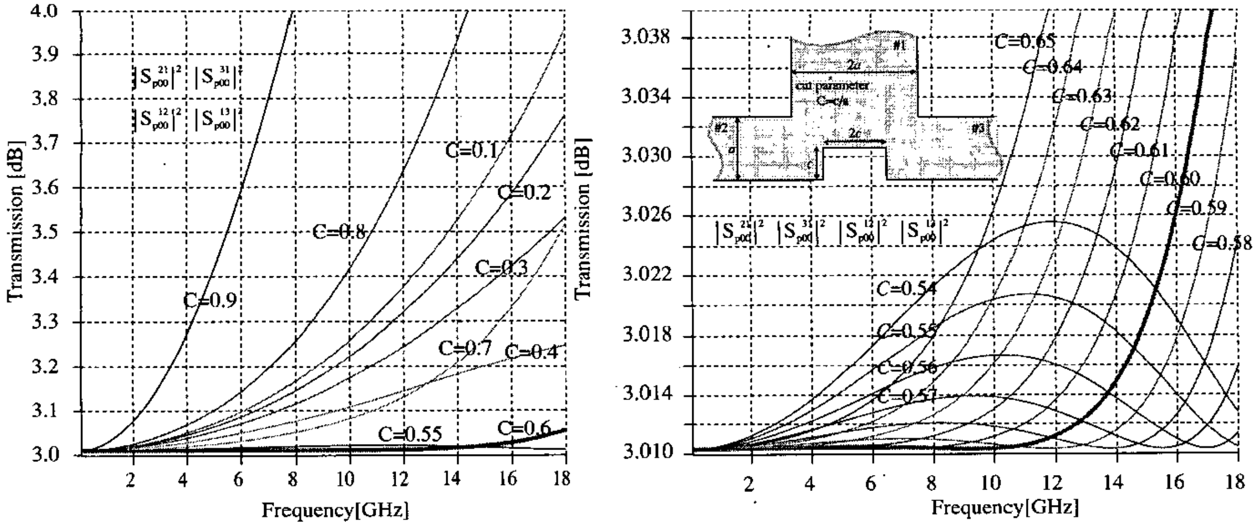


Fig.5 Transmission vs frequency characteristics for planar T-junction with rectangular cut
and its detail about C=0.6 (cut parameter C=c/a)