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COMPUTER MEMORY REDUCTION BY INTRODUCING CUBIC BOUNDARY ELEMENT IN SOUND FIELD SIMULATION

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1. INTRODUCTION

To reduce computer storage requirement in numerical analysis of a sound field, we introduce higher-order element into our BE computer program which contains a sub-region frontal technique and a partial Gauss-Jordan elimination for the nodal state variables of the target regions [1]. We will give some test results on numerical attenuation in a loss-free stiff-wall duct and on the sound absorption of a slit resonator with wall visco-thermal dissipation [2]. We describe here on two dimensional cases.

2. NUMERICAL ANALYSIS METHOD

BE for Acoustic-wave Mode. Dividing a given field into sub-regions and each sub-region surface S into N surface patches, we denote the surface of a patch ℓ by S_ℓ. As field valuables of the acoustic-wave mode we employ sound pressure p and its outward normal gradient q on the surface and introduce a discontinuous cubic element with 4 nodes, as shown in Fig.1, to approximate the distribution of the field variables over each surface patch S_ℓ in terms of their nodal values $p_k = p(\xi = \xi_k)$ and $q_k = q(\xi = \xi_k)$ in which $k = 1,2,3,4$, as

$$p(\xi) = \sum_{k=1}^{4} \psi_k(\xi) p_k, \quad q(\xi) = \sum_{k=1}^{4} \psi_k(\xi) q_k \tag{1}$$

where ξ is the natural coordinate taken along a surface patch S_ℓ and

$$\left.\begin{matrix}\psi_1(\xi)\\ \psi_4(\xi)\end{matrix}\right\} = \frac{-1}{48}(1-4\xi)(1+4\xi)(3\mp 4\xi), \quad \left.\begin{matrix}\psi_2(\xi)\\ \psi_3(\xi)\end{matrix}\right\} = \frac{1}{16}(1\mp 4\xi)(3-4\xi)(3+4\xi)$$

$$\left.\begin{matrix}\xi_1\\ \xi_4\end{matrix}\right\} = \mp\frac{3}{4}, \quad \left.\begin{matrix}\xi_2\\ \xi_3\end{matrix}\right\} = \mp\frac{1}{4}. \tag{2}$$

For each sub-region, applying the Kirchhoff-Helmholtz integral theorem, we have a linear equation system (3), as

$$0=-\alpha_i p_i+\sum_{\ell=1}^{N}\int_{S_\ell}(-q^*p+p^*q)ds=\sum_{\ell=1}^{N}\sum_{k=1}^{4}(H_{ij}p_j+G_{ij}q_j) \tag{3}$$

where $i=1,2,\cdots,4N$, $j=k+4(\ell-1)$, $p^*=-(j/4)H_0^{(2)}(r_{ij}\omega/c)$ stands for the free space Green's function of the two dimensional inhomogeneous Helmholtz equation, in which $H_0^{(2)}()$ is the Hankel function of the second kind and of zero order, q^* is the outward normal gradient of p^* on the surface patch S_ℓ, ω is the angular frequency, c is the speed of sound, r_{ij} is the distance between points i and j. $\alpha_i=1/2$ since we take the point i on a smooth part of the surface S. The integrals along the surface patch S_ℓ is written as

$$H_{ij}=-\alpha_i\delta_{ij}-\int_{-1}^{1}q^*(\xi)\psi_j(\xi)A(\xi)d\xi,\ G_{ij}=\int_{-1}^{1}p^*(\xi)\psi_j(\xi)A(\xi)d\xi \tag{4}$$

in which $A(\xi)=ds/d\xi$, and δ_{ij} is the Kronecker delta. To represent the shape of each surface patch, we employ a quadrilateral element with 3 nodes.

Boundary Conditions with Visco-Thermal Admittance. We assume that acoustic properties of the original boundary are described locally by the driving pressure f and the admittance β which contains the effective visco-thermal admittance to match the acoustic-wave mode to the thermal- and shear-wave modes as well as the wall admittance [2], as

$$q=-j\omega\rho\beta(p-f)\quad,\ \beta=\frac{1}{z}+(1+j)\{\frac{R_h}{\rho^2c^2}-\frac{R_v\nabla_{\tan}^2}{\rho^2\omega^2}\} \tag{5}$$

where $q/(-j\omega\rho)$ is the outward normal velocity of the acoustic mode, z is the specific acoustic impedance of the wall. $R_v/\rho c=\omega d_v/2c$, $R_h/\rho c\ \omega(\gamma-1)d_h/2c$, where $d_v=\sqrt{2\mu/\omega\rho}$ and $d_h=\sqrt{2\kappa/\omega\rho c_p}$, μ and κ are the viscosity and the thermal conductivity of the air, c_p and γ are the specific-heat coefficient at constant pressure and the specific-heat ratio of the air, respectively. The tangential Laplacian $\nabla_{\tan}^2$ stands for the second derivatives with respect to the coordinate tangential to the boundary surface, and $\nabla_{\tan}^2 p$ is given as

$$\nabla_{\tan}^2 p=\frac{d^2p}{ds^2}=\frac{1}{A^2}\sum_{k=1}^{4}\sigma_k(\xi)p_k,\ \left.\begin{matrix}\sigma_1(\xi)\\ \sigma_4(\xi)\end{matrix}\right\}=\mp 8\xi+2,\ \left.\begin{matrix}\sigma_2(\xi)\\ \sigma_3(\xi)\end{matrix}\right\}=\pm 24\xi-2 \tag{6}$$

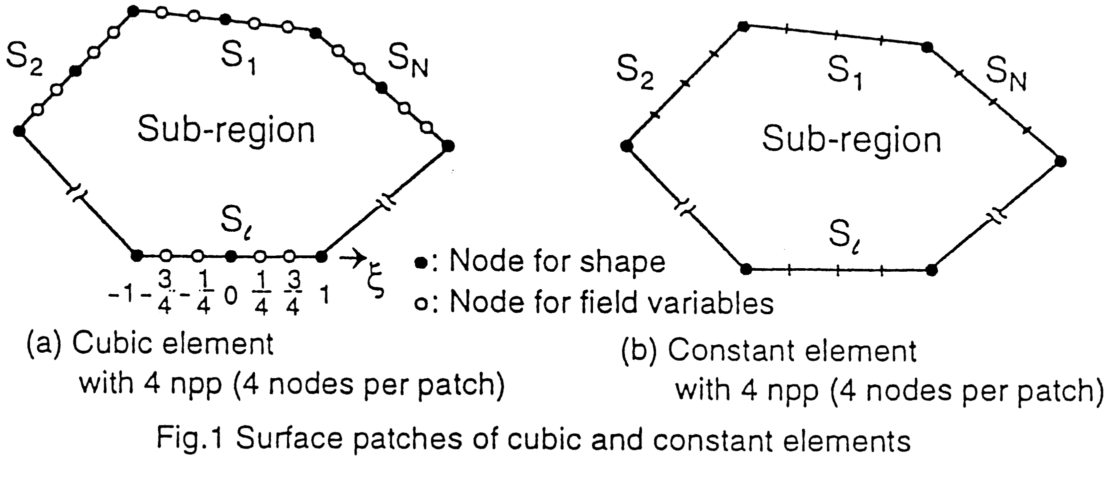


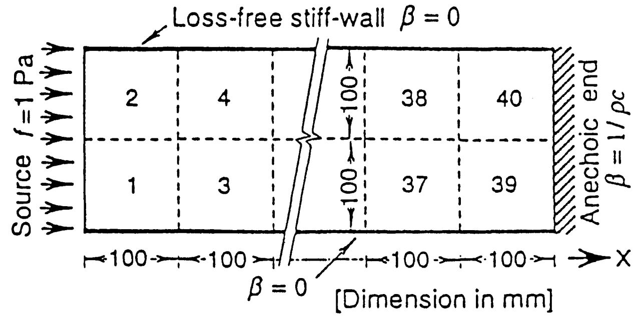
Fig.1 Surface patches of cubic and constant elements

In case of the constant element, $\nabla^2_{\tan}p$ of a nodal point was approximated in a finite difference expression by using the neighboring nodal pressures in a surface patch [2].

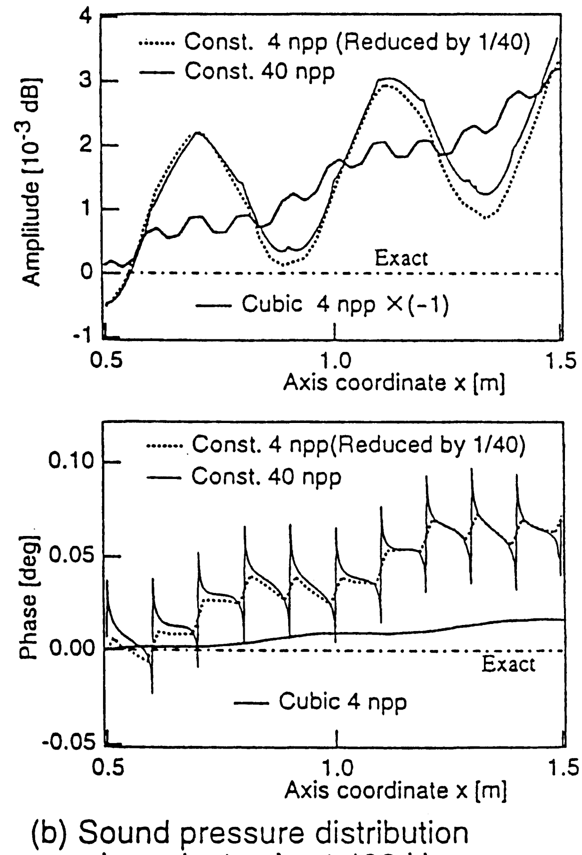
Reduction Methods of Computer Storage. We have employed the frontal technique of equation assembly and reduction to solve for the interface variables combining the set of equations (3) for each sub-region together with compatibility and equilibrium conditions between their common interfaces. This technique contributes to minimize core storage as well as present introduction of the higher order element. In addition, restricting the target to the partial regions of interest instead of whole field, we have introduced the Gauss-Jordan elimination for the target unknowns in the Gauss elimination process to omit buck-up storage for the buck-substitution process.

3. NUMERICAL TESTS

To investigate numerical attenuation caused by coarse discretization, tests were conducted on the sound field of a loss-free stiff-wall duct as shown in Fig.2(a). The duct is 0.2m width and 2m long with sound source at one end and anechoic termination at the opposite end so that only a plane wave in the positive axial direction exists below cross-mode cut-on frequency (850 Hz). The sound pressure distribution (normalized by values at x=0.5 m) of the target section at 400 Hz are shown in Fig. 2(b). Numerical error in amplitude and phase by cubic element of 4 npp (which stands for nodes per patch) are about -



(a) Numerical model

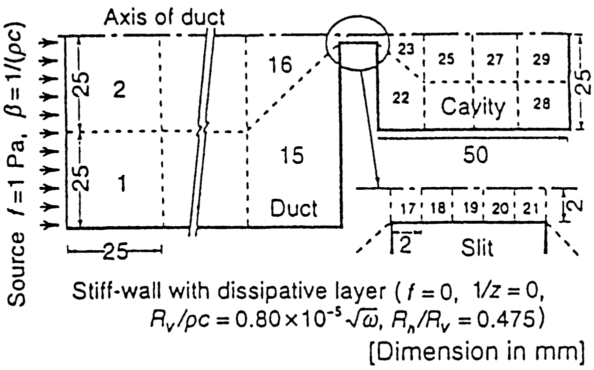


(b) Sound pressure distribution along duct axis at 400 Hz
(Normalized by values at x=0.5m)

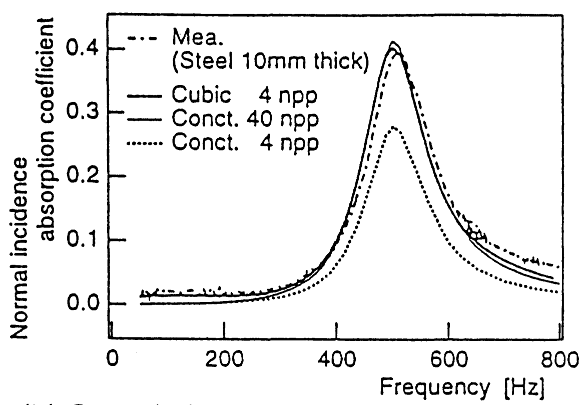
Fig.2 Numerical attenuattion in a loss-free stiff-wall duct

-0.003 dB/m and 0.02 deg/m, respectively. Those by constant element fluctuate large, and about 0.003 dB/m and 0.08 deg/m when 40 npp is used, while about 0.003×40 dB/m and 0.07×40 deg/m when 4 npp is used. Cubic element of 4 npp and constant element of 40 npp is similar in precision for amplitude. This indicates that cubic element requires less computer storage by about 1/100 compare to constant element at a given order of precision, though the frequency dependence must be investigated further.

To confirm the effectiveness of introduction of cubic element to the evaluation of the tangential Laplacian for wall visco-thermal dissipation, a test was carried out on a slit resonator as illustrated in Fig. 3(a). Fig.3(b) shows the results on sound absorption. Here again, cubic element of 4 npp gives similar precision to constant element of 40 npp.



(a) Numerical model
(Half of field with respect to the duct axis is modeled)



(b) Sound absorption

Fig.3 Sound absorption of a slit resonator

4. CONCLUSION

To reduce computer storage in numerical prediction of a sound field, cubic element has introduced. Tests has been carried out on numerical attenuation in a loss-free stiff-wall duct and on sound absorption of a slit resonator with wall visco-thermal dissipation. Consequently cubic element has reduced computer memory requirement by roughly 1/100 compare to constant element at a given order of precision in these two dimensional cases.

REFERENCES

1. M. Terao *et al*, "A numerical analysis of sound field of a long space by a sub-region coupling approach", *Inter-noise 96*, Liverpool, 1996.
2. M. Terao *et al*, "A boundary element approach to determine acoustic properties of Helmholtz resonators", *Inter-noise 95*, Newport beach, 1995.