

**A NUMERICAL ANALYSIS OF SOUND FIELD OF A LONG SPACE BY A SUB-REGION COUPLING APPROACH**

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**1. INTRODUCTION**

To reduce computer storage requirement in numerical analysis of a large sound field at audible frequencies, we employ a sub-region technique and we restrict the target of prediction to a partial field instead of the whole field by introducing the Gauss-Jordan elimination for the partial equations corresponding to the state variables (typically, sound pressures and particle velocities) of the target regions in the direct Gauss elimination process. Some numerical test results will be given for a partial field in a 6mx10mx20m rectangular space at 500 Hz and for lined duct attenuation of a 10mx0.05mx0.25m duct at frequencies above cross-mode cut-on on.

**2. NUMERICAL METHOD**

**Sub-region BE Modeling**

The space of interest is divided into sub-regions by taking interfaces as shown in Fig. 1. We employed the discontinuous cubic element with 4x4 node points to approximate each state variable distribution on each sub-region surface. For each region, for region I for instance, we have a linear equation system (1), applying the Kirchhoff-Helmholtz integral theorem as

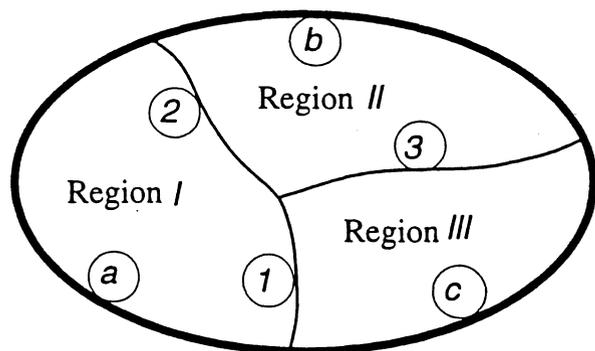


Fig.1 Sub-region BE modeling.

$$g_{ia}u_a + h_{ia}p_a + \sum_{j=1}^2 (g_{ij}u_j + h_{ij}p_j) = 0 \quad , i = a, 1, 2 \quad (1)$$

where  $p_i$  and  $u_i$  denote the sound pressure and particle velocity at node  $i$ . The coefficients  $g_{ij}$  and  $h_{ij}$  are given by applying the free field Green's function of Helmholtz equation to the geometry between point  $i$  and surface  $j$ .

On the boundary of the space, for instance, on the boundary surface  $a$  of the sub-region  $I$ , the boundary condition is generally given as

$$p_a = f_a + Z_a u_a \quad (2)$$

where  $f_a$  denotes a row vector containing 16 driving pressures acting on the boundary,  $Z_a$  is a square matrix containing 16x16 impedances of the boundary,  $u_i$  and  $p_i$  are column vectors, and,  $g_{ij}$  and  $h_{ij}$  are row vectors of 16 entries.

### Computer storage reduction in coupling of sub-regions

Elimination of  $p_a$  and  $u_a$  from equations (1) and (2), yields the expression (3) relating the interface variables ( $p_i$  and  $u_i$ ) and the characteristics ( $G_{ij}$  and  $H_{ij}$ ) of region  $I$  as

$$\sum_{j=1}^2 (G_{ij} u_j + H_{ij} p_j) = F_i \quad , i = 1, 2 \quad (3)$$

We have the same expression (3) between the interface variables for the other sub-regions  $II$  with  $i=2,3$  and  $III$  with  $i=3,1$ .

To minimize core storage requirement in solving for the interface variables of the set of equations (3) for each sub-region together with compatibility and equilibrium conditions between their common interfaces, the frontal technique of equation assembly and reduction is effective. In this technique, the sub-regions are considered each in turn according to a prescribed order. Whenever a new sub-region equation set (3) is read from a disc file into the core, the variables for the common interfaces between the new region and the region corresponding to existing interface equations in the core are eliminated. The equations used for this elimination of the common interface variables are freed from the core for the next sub-region equation set and stored away on a back-up disc file for the buck-substitution process. However the file size of this buck-up disc becomes huge and will not be available even by high end computer systems of today when the space of interest is as large as a music hall. We restrict the target to some partial regions or finite points instead of whole of such a large field. To omit the buck-substitution process, we introduce the Gauss-Jordan elimination only for the equations corresponding to the target unknowns in the Gauss elimination process.

## 3. NUMERICAL TESTS

### High-frequency Attenuation of Lined Ducts

Numerical tests were conducted on the sound field of a 10m long, 0.05m high and 0.25m width duct with sound absorbing liner on its one side as shown in Fig. 2. Each cube cell (0.05m x 0,05m x 0,05m) of BE sub-region

has 6 surfaces of discontinuous cube (4x4 nodes) elements. On  $y=0.25\text{m}$  surface normal impedance of 0.05m thick polyurethane foam with flow resistance of 8500 Pa s/m<sup>2</sup> was given corresponding to C. Wassilieff [1]. The locally reacting model was employed for simplicity. On the other surfaces assumed to be stiff. The  $x=0$  plane was evenly driven at 1Pa. Target section was chosen on the plane ( $z=0, y=0\text{m} - 0.25\text{m}, x=5\text{m} - 7\text{m}$ ). The sound pressure and intensity distributions of this plane are shown in Fig. 3. The duct length of 10m is not enough to suppress evanescent modes generated from both ends. The attenuation of the lined duct taken from the pressure distribution of this target section is shown in Fig. 4. The theoretical of bulk reacting lining model [1] is shown for comparison.

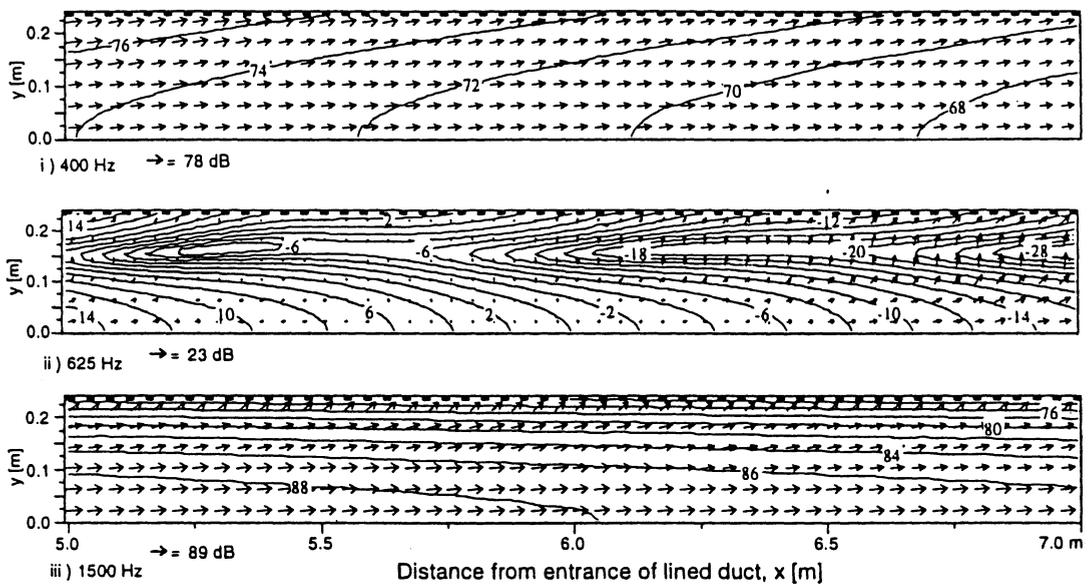


Fig. 3. Sound pressure and intensity distributions (in dB) on target plane of Fig. 2.

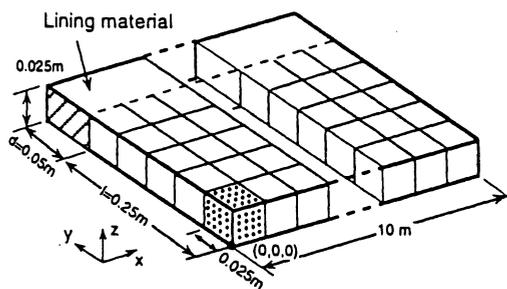


Fig. 2. Sub-region BE model of lined duct tested.

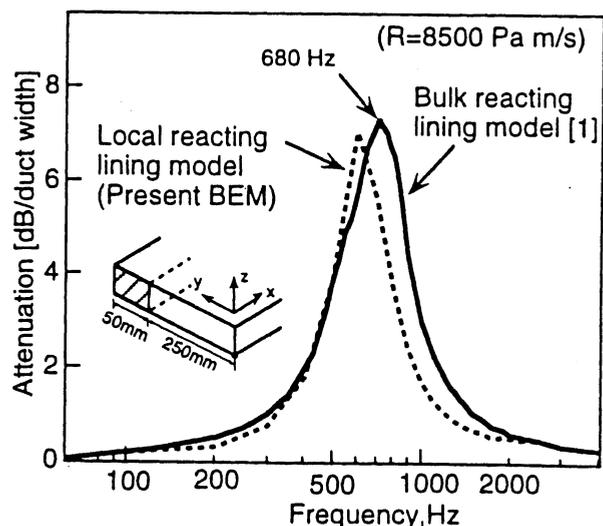


Fig. 4. Lined duct attenuation

Considering less cross-mode contribution in our case, this degree of agreement is encouraging us to develop further this approach of computer storage reduction.

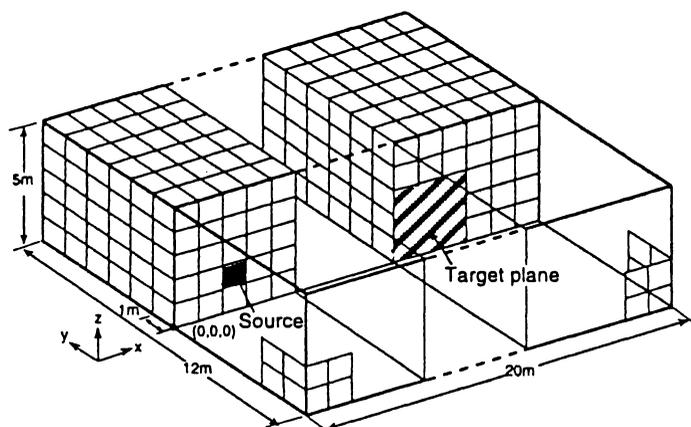


Fig. 5. Sub-region BE model of a large space.

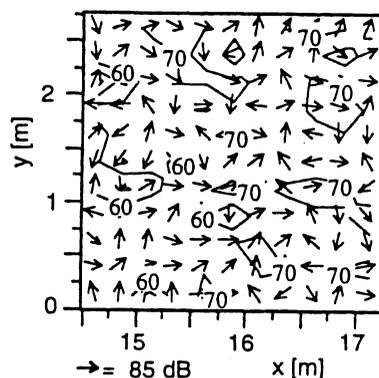


Fig. 6. Sound pressure and intensity distributions (in dB) on target plane in Fig. 5.

### Mid-frequency Sound Field in a Large Space

Fig. 5. shows one of the models tested for a large space case. The dimension of each cube cell taken was  $1\text{m} \times 1\text{m} \times 1\text{m}$ . A sound source is located at point  $(3\text{m}, 0, 1.5\text{m})$ . The normal impedance of  $z/\rho c = 0.069 - j1.52$ , corresponding to the duct liner above, was given on the ceiling and back wall. The other walls and floor was assumed to be stiff. The target surface of prediction was taken on the plane  $(y=0, z=0 - 3\text{m}, x=14.5 - 16.5\text{m})$  Fig. 6. shows the pressure distribution on this target surface. We compared the pressure and velocity distribution, the memory sizes and computation times required for this space model, by employing double precision calculations, between the partial Gauss-Jordan elimination and the total Gauss elimination method. Both results for these distributions coincide at least up to 10 most significant decimals. Although the computation times were similar, the computer storage required are approximately 0.14 and 5.3 GB, respectively, for the partial Gauss-Jordan and the total Gauss elimination method.

## 4. CONCLUSION

To reduce the computer storage in the numerical prediction of a sound field with large dimension times frequency, a partial Gauss-Jordan elimination method has been introduced and confirmed effective by numerical test on a lined duct at cross-mode frequencies and on a rectangular space as large as a mid-size conference room at 500 Hz.

### References

- 1.C. Wassilieff, J. Acoust, Soc, Am., 114, 239(1987). Figure 4 (a).