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A one-dimensional wave model for numerical analyses of ventilation openings with Helmholtz resonator attachments

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Abstract [380] A ventilation opening becomes a primary sound transmission path for a room when the room employs double glazing windows for sound insulation. Especially it is difficult to suppress the noise component that coincides with the lowest resonance frequency of the longitudinal modes of the opening. For this mode, Helmholtz resonator attachments can be effective while mufflers made of sound absorbing materials work only a little. However in designing a Helmholtz resonator attachment to a ventilation opening, a series of trial and error simulations are indispensable because resonators have reactive nature. Numerical analyses based on one-dimensional wave models are preferable for the repetitious optimal design stage. However, for a resonator array of very short longitudinal intervals of a few centimeters, effectiveness of one-dimensional wave models has not been confirmed so far. Several prototypes of Helmholtz resonator attachments were investigated here. Their performances were determined by conducting experimental tests, numerical simulations by a one-dimensional plane wave model and those by a three-dimensional BEM. Consequently, for an transmission loss curve (for 100 discrete frequencies) of an opening of less than a half meter long, the numerical analysis by the three-dimensional BEM took a week while that by the one-dimensional plane wave model performed instantaneously. Nevertheless the results by the one-dimensional wave model agree excellently with those by the three-dimensional BEM, and agree fairly well with those by the experiments.

1 INTRODUCTION

For a room where double glazing windows must be employed for sound insulation, ventilation openings may often become primary sound transmission paths. Especially it is difficult to suppress the noise component that coincides with the lowest resonance frequency of the longitudinal modes of the opening. For this mode, Helmholtz resonator attachments can be effective while silencers made of sound absorbing materials hardly work. However in designing a Helmholtz resonator attachment to a ventilation opening, a series of trial and error simulations are indispensable because resonators have reactive nature. Numerical analyses based on one-dimensional plane wave models are preferable for the repetitious optimal design stage. However, for a resonator array of very short longitudinal intervals of a few centimeters, effectiveness of one-dimensional plane wave models has not been confirmed so far. Several prototypes of Helmholtz resonator attachments were made and investigated here. Their performances were determined by conducting experimental tests, numerical simulations by a one-dimensional wave model and those by a three-dimensional BEM. Comparison was made with the results by the one-dimensional plane wave model, by the three-dimensional BEM and by the experiments.

2 A RESONATOR ARRAY ATTACHMENT FOR A VENTILATION OPENING

We consider a Helmholtz resonator array attachment (HA) to suppress the noise component that coincides with the lowest resonance frequency of the longitudinal modes of a ventilation opening (of cross-sectional area $S_\mathrm{D} = 0.05^2\pi$) of a wall of thickness 200 mm as shown in Figure 1. The resonator array attachment, in this case, is composed of 3 Helmholtz resonators (HR1, HR2, and HR3) as side branches. In the figure, the superscript (n) represents for the n th resonator. $V_\mathrm{H}^{(n)}$, $S_\mathrm{H}^{(n)}$ and $l_\mathrm{H}^{(n)}$ denote the volume, the aperture sectional area and the aperture length, respectively, of the n th Helmholtz resonator.

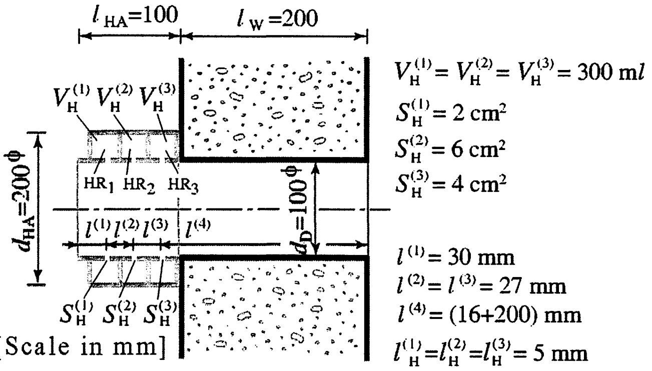


Figure 1 A Helmholtz resonator array attachment to a ventilation opening.

3 ONE-DIMENSIONAL PLANE WAVE ANALYSIS OF A RESONATOR ARRAY

3.1 Acoustic Impedance of a Helmholtz Resonator

For one of the duct section containing a Helmholtz resonator as a side branch, we employ a one-dimensional plane wave model as shown in figure 2. In those one-dimensional wave analyses we use the acoustic impedance of a resonator $Z_\mathrm{H} = p_\mathrm{H} / U_\mathrm{H}$ defined as

$$Z_\mathrm{H} = R_\mathrm{H} + j\left\{(l_\mathrm{A} + \Delta l_\mathrm{out} + \Delta l_\mathrm{inn})\omega\rho / S_\mathrm{H} - \rho c^2 / \omega V_\mathrm{H}\right\}. \quad (1)$$

where U_H denotes the complex amplitude of the volume velocity flowing into the resonator, p_H is the complex pressure amplitude at the duct junction with the resonator, ω is the radian frequency, ρ is the density of the air, c is sound speed in the air, S_H and l_H are the sectional area and the length, respectively, of the resonator aperture, V_H is the volume of the cavity, R_H stands for the acoustic resistance of the resonator, and Δl_inn and Δl_out are the inner and outer orifice-correction of the resonator aperture. To give an explicit expression for the outer orifice mass end correction Δl_out of a resonator attached to the sidewall of a circular duct, we used the empirical formula [1] which is obtained by a series of full wave simulations by the 3-D BEM.

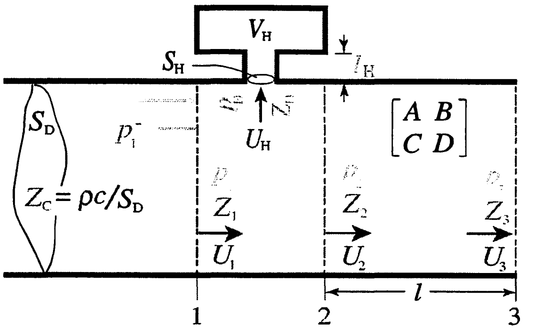


Figure 2 A plane wave model of a duct section containing a Helmholtz resonator as a side branch.

3.2 One-Dimensional Model of a Duct Section Containing a Helmholtz Resonator

In the one-dimensional simulation by using the plane wave model for a duct section with a Helmholtz resonator, we take ports (interfaces) $i = 1$, 2 and 3 as shown in Figure 2. For every port i, the sound pressure amplitude and the volume velocity amplitude p_i and U_i, respectively, are related by the acoustic impedance Z_i as $p_i = Z_i U_i$.

On the port 1, the sound pressure amplitude p_1, the volume velocity amplitude U_1, the incident wave pressure amplitude p_1^+, and the reflected wave pressure amplitude p_1^- have relationships as

$$p_1 = p_1^+ + p_1^-,\ Z_c U_1 = p_1^+ - p_1^-,\ Z_c = \rho c / S_D \qquad \text{(2a, b, c)}$$

where Z_c denotes the characteristic acoustic impedance. These yield a relationship $2p_1^+ = p_1 + Z_c U_1$ and, taking $p_1 = Z_1 U_1$ into consideration, we have

$$2p_1^+ = (Z_1 + Z_c)U_1. \qquad (3)$$

Between the ports 1 and 2, the pressure and volume-velocity amplitudes have relationships as

$$p_1 = p_2 = p_H = Z_H U_H,\ U_1 = U_H + U_2 \qquad \text{(4a, b)}$$

Taking $p_1 = Z_1 U_1$ and $p_2 = Z_2 U_2$ into account, we have

$$Z_1 U_1 = Z_2 U_2 = Z_H U_H \qquad (5)$$

For a two-port element 2-3 between the ports 2 and 3, the pressure and volume-velocity amplitudes are related in terms of the fundamental matrix, for instance, as

$$p_2 = Ap_3 + BU_3,\ U_2 = Cp_3 + DU_3 \qquad \text{(6a, b)}$$

where A, B, C and D are the four terminal constants. In case of a straight duct of length l, for instance, they are represented as $A = D = \cos kl$, $B = jZ_c \sin kl$, $C = jZ_c^{-1} \sin kl$.

Taking $p_3 = Z_3 U_3$ into account, Eq. (3) and (6) yield

$$1/Z_1 = 1/Z_H + 1/Z_2,\ Z_2 = (AZ_3 + B)/(CZ_3 + D). \qquad \text{(7a, b)}$$

3.3 Dissipation and Transmission Factors of a Helmholtz Resonator Array

Figure 3 shows a duct resonator array composed of N duct sections each of which contains a Helmholtz resonator. By representing the ports as $1^{(n)}$, $2^{(n)}$ and $3^{(n)}$ for a duct section n ($n = 1, 2, \cdots, N$), we can directly apply Eq. (7) as

$$1/Z_1^{(n)} = 1/Z_H^{(n)} + 1/Z_2^{(n)},\ Z_2^{(n)} = (A^{(n)}Z_3^{(n)} + B^{(n)})/(C^{(n)}Z_3^{(n)} + D^{(n)}). \qquad \text{(8a, b)}$$

At an interface between adjacent sections, n and $n-1$, we can use the relationships

$$p_1^{(n)} = p_3^{(n-1)},\ U_1^{(n)} = U_3^{(n-1)},\ \text{and}\ Z_1^{(n)} = Z_3^{(n-1)}. \tag{9}$$

When the termination impedance $Z_3^{(N)}$ of N th (the last) duct section is given, the impedances $Z_3^{(n)}$, $Z_2^{(n)}$ and $Z_1^{(n)}$ of every duct section n can be determined by using Eqs. (8) for $n = N, N-1, \cdots, 2, 1$ in tern.

The dissipation factor of a Helmholtz resonator is defined as the ratio of the dissipated sound power $P_{\mathrm{H}}^{(i)} = R_{\mathrm{H}}^{(i)} \left|U_{\mathrm{H}}^{(i)}\right|^2 / 2$ to the incident sound power $P_1^+ = \left|p_1^+\right|^2 / 2Z_{\mathrm{c}}$, i.e., the dissipation factor of i th resonator $\delta_{\mathrm{H}}^{(i)}$, is written as

$$\delta_{\mathrm{H}}^{(i)} = Z_{\mathrm{c}} R_{\mathrm{H}}^{(i)} \left|U_{\mathrm{H}}^{(i)} / p_1^+\right|^2 = 4 Z_{\mathrm{c}} R_{\mathrm{H}}^{(i)} \left|U_1 / 2p_1^+\right|^2 \left|U_{\mathrm{H}}^{(i)} / U_1^{(i)}\right|^2 \prod_{n-1}^{i-1} \left|U_3^{(n)} / U_1^{(n)}\right|^2 . \tag{10}$$

For a duct section between the ports $1 \equiv 1^{(1)}$ and $3^{(i)}$, the transmission factor $\tau^{(i)}$ is defined as the ratio of the transmitted sound power $P_3^{(i)} = R_3^{(i)} \left|U_3^{(i)}\right|^2 / 2$ to the incident sound power, i.e.,

$$\tau^{(i)} = Z_{\mathrm{c}} R_3^{(i)} \left|U_3^{(i)} / p_1^+\right|^2 = 4 Z_{\mathrm{c}} R_3^{(i)} \left|U_1 / 2p_1^+\right|^2 \prod_{n=1}^{i} \left|U_3^{(n)} / U_1^{(n)}\right|^2 . \tag{11}$$

These can be determined by using Eq. (3), i.e., $2p_1^+ / U_1 = Z_{\mathrm{c}} + Z_1$ and Eq. (4), and the following relationships derived from Eqs. (5) and (6b);

$$U_{\mathrm{H}}^{(i)} / U_1^{(i)} = Z_{\mathrm{H}}^{(i)} / Z_1^{(i)} \ \text{and}\ U_1^{(n)} / U_3^{(n)} = (Z_{\mathrm{H}}^{(n)} + Z_2^{(n)})(C^{(n)} Z_3^{(n)} + D^{(n)}) / Z_{\mathrm{H}}^{(n)} . \tag{12a, b}$$

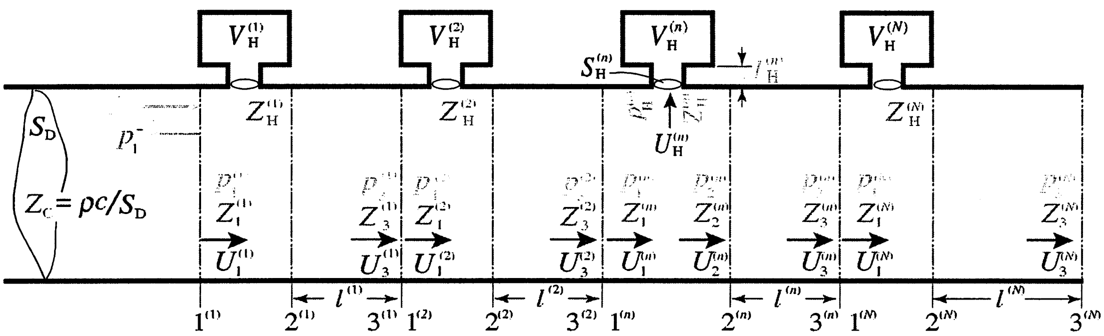


Figure 3 A Helmholtz resonator array of a duct

3.4 Application of Duct Resonator Array Model to a Ventilation Opening

To specify the sound insulation performance of a silencer of a ventilation opening, we introduce a standardized transmission loss, R_{s}, which is similar to the normalized sound transmission loss [2]. R_{s} is defined as the difference between the sound pressure level of the incidence wave (whose acoustic intensity equal to $\left|p_1^+\right|^2 / 2\rho c$) and the sound power level of the transmitted wave ($P_3^{(i)} = R_3^{(i)} \left|U_3^{(i)}\right|^2 / 2$). To determine R_{s} for the resonator array containing $3 (= N-1)$ resonators applying Eq. (11), we represent R_{s} as

$$R_{\mathrm{s}} = -10 \log \tau^{(N)}, \tag{13}$$

and take $N = 4$, $Z_{\mathrm{H}}^{(1)} \to \infty$, and substitute Z_{c} by Z_{rad} for incidence port [cf. Eq.(3) and Figure 3].

4 EXPERIMENTAL SET UP

To confirm the effectiveness of the numerical simulation, experiments were carried out on the set up as shown in Figure 4. A wall of 200 mm thick, 2 m height and 4m width was installed in an anechoic room. A ventilation opening was located at the center of the wall. A 5mm thick PVC duct of 250 mm diameter was employed to introduce the normal incidence sound from a loudspeaker, and to avoid the flanking sound transmission. The cut-on frequency of the first (0, 1) cross mode of the 250 mm diameter duct is 810 Hz. The impedance-tube two-microphone method was employed to extract the incidence sound pressure from the in-duct pressure field.

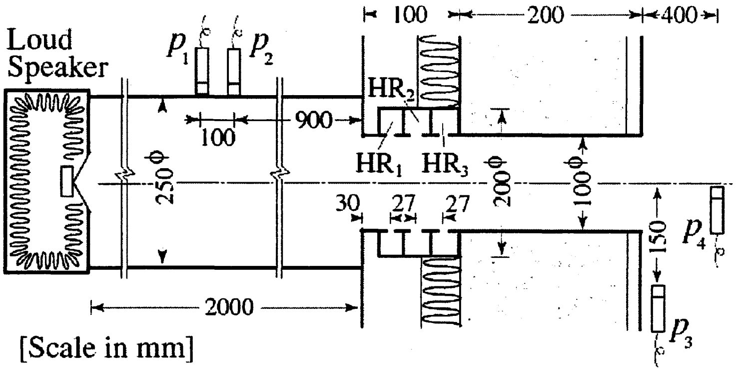


Figure 4 Experimental set up to measure acoustic transmission loss of a ventilation opening

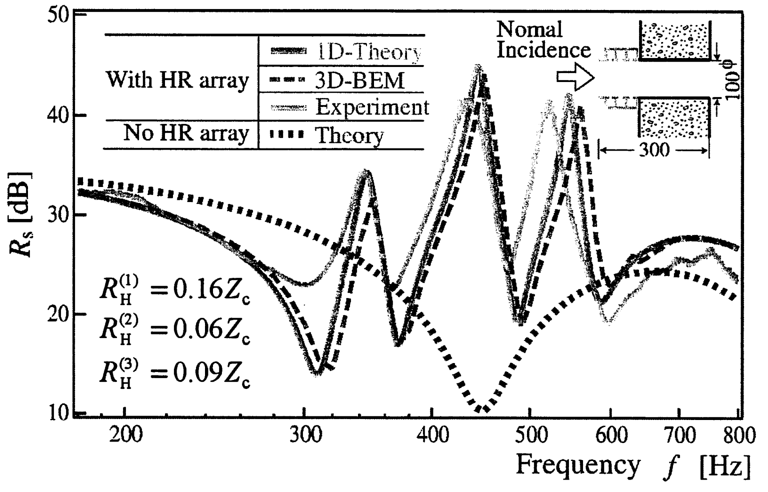
5 APPLICATION OF 1-D WAVE MODELTO A VENTILATION OPENING

By conducting a series of trial and error simulations using the one-dimensional plane wave model, Eq. (13), the optimal Helmholtz resonator attachment was sought for to suppress the noise component (440Hz) that coincides with the lowest resonance frequency of the longitudinal modes of the ventilation opening. For typical Helmholtz resonator attachments, the transmission losses of the ventilation opening with the attachments were determined by conducting experimental tests and numerical simulations by a three-dimensional BEM as well as numerical simulations by the one-dimensional plane wave model.

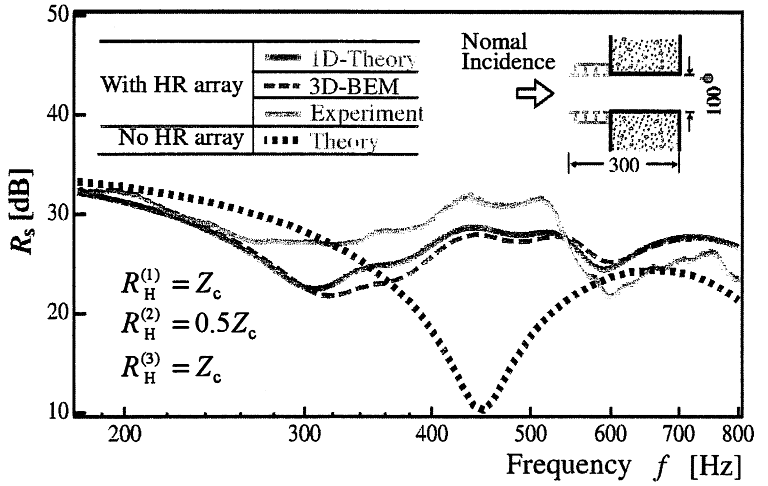
Figure 5 shows two typical results. For an standardized transmission loss curve (for 100 discrete frequencies at 10Hz step) of a ventilation opening of less than a half meter long, the numerical analysis by the three-dimensional BEM took a week while that by the one-dimensional wave model performed instantaneously. Nevertheless the results by the one-dimensional wave model agree excellently with those by the three-dimensional BEM, and agree fairly well with those by the experiments. This confirms the effectiveness of the one-dimensional model on the subject. The resonator array attachment as shown in Figure 5(b) improves the transmission loss by nearly 20 dB at the lowest resonance frequency of the longitudinal modes of the ventilation opening.

6 CONCLUSION

To suppress the noise component that coincides with the lowest resonance frequency of the longitudinal modes of a ventilation opening, effectiveness of a one-dimensional plane wave model for the optimizing process of Helmholtz resonator attachments was investigated. The performances of a ventilation opening containing several Helmholtz resonator attachments were determined by using this model. Typical of them were determined also by experimental tests and numerical simulations by a three-dimensional BEM to make comparison. Consequently, to obtain a transmission loss curve of a ventilation opening configuration, the numerical analysis by the three-dimensional BEM took a week while that by the one-dimensional plane wave model performed instantaneously. Nevertheless the results by the one-dimensional wave model agree excellently with those by the three-dimensional BEM, and agree fairly well with those by the experiments.



(a) When resonator resistances are small compare to characteristic impedance of duct



(b) When resonator resistances are nearly coincident with characteristic impedance of duct

Figure 5 Effectiveness of 1-D model for a ventilation aperture containing Helmholtz resonators

REFERENCES

[1] M. Terao, H. Sekine, S. Ohkawa and D. Hayashi, On one-dimensional sound analysis of a duct network with Helmholtz resonators, Proc. of inter-noise 2002, 2002, N439.

[2] H. Ohkawa, H. Tachibana and M. Koyasu, Sound transmission characteristics of building elements with small openings for ventilation purposes, Proc. of inter-noise 84,1984,pp603-606.