inter-noise 2002

The 2002 International Congress and Exposition on Noise Control Engineering
Dearborn, MI, USA. August 19-21, 2002

On One-dimensional Sound Analysis of a Duct Network with Helmholtz Resonators

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Abstract

The outer orifice correction for Helmholtz resonators attached to the sidewall of circular ducts was studied. For the outer orifice correction when the axis direction of the orifice coincides with that of the duct, explicit expressions were given by Ingard and Rschevkin. But their application to duct sections with duct-sidewall resonators is beyond their premise. An explicit expression of the outer orifice correction for duct-sidewall resonators was derived by conducting three-dimensional boundary-element analyses. Application of this outer orifice correction improves significantly the accuracy of the one-dimensional wave analysis for the acoustic properties of duct sections which have sidewall resonators.

1. Introduction

In designing Helmholtz resonators for a duct network, acoustical simulations are indispensable because the resonators introduce acoustically abrupt discontinuities at their apertures in the duct field. To obtain one transmission-loss curve of a resonator-attached duct-section of several meters long, the three-dimensional boundary element analysis takes half a day while one-dimensional plane-wave analysis perform in a minute. The latter analysis is preferable at the stage of trial and error optimization of the resonator design parameters.

However the one-dimensional wave analysis requires estimation of the radiation impedance for the outside field of each resonator aperture. In this study, for simplicity, estimation of the radiation reactance is focused on in terms of the outer orifice correction Δl_{out}. For Δl_{out} explicit expressions were given by Ingard and Rschevkin [1]. But this correction may vary with the angle θ_{A} between the duct axis and the resonator aperture axis. Ingard and Rschevkin's expression are Δl_{out} for

$\theta_A = 0°$. In this paper an explicit expression of Δl_{out} for side-branch resonators attached to duct walls ($\theta_A = 90°$) is derived by conducting three-dimensional boundary-element analyses. Application of this correction Δl_{out} for $\theta_A = 90°$ improves significantly the effectiveness of the one-dimensional analysis in the transmission loss prediction of duct sections containing sidewall resonators.

2. Acoustic Impedance of Helmholtz Resonator

We consider a straight duct section (of cross-sectional area S_D) that has a Helmholtz resonator attached to the duct sidewall as shown in Figure 1.

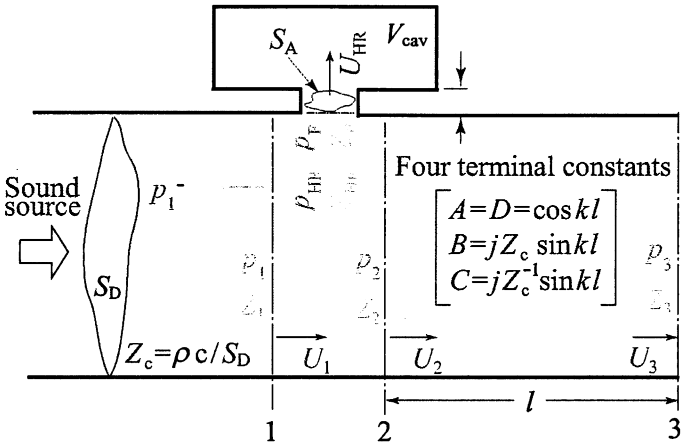

Figure 1 A plane wave model for a Helmholtz resonator as a side branch of a straight duct section.

The full wave analyses use the inner acoustic impedance Z_F defined as $Z_F = p_F / U_{HR}$ where U_{HR} denotes the complex amplitude of the volume velocity flowing into the resonator, p_F is the complex pressure amplitude just outside of the aperture of the resonator, and

$$Z_F = R_F + j\left\{(l_A + \Delta l_{inn})\omega\rho / S_A - \rho c^2 / \omega V_{cav}\right\} \tag{1}$$

where R_F stands for the acoustic resistance of the resonator, ω is the radian frequency, ρ is the density of the air, c is sound speed in the air, S_A and l_A are the sectional area and the length, respectively, of the aperture, V_{cav} is the volume of the cavity, and Δl_{inn} is the inner orifice-correction, i.e., the mass end correction for the cavity side of the aperture.

On the other hand, one-dimensional wave analyses use the total acoustic impedance of a resonator $Z_{HR} = p_{HR} / U_{HR}$ where p_{HR} denotes the complex pressure amplitude at the junction with the resonator, and is defined as $p_{HR} = p_F + Z_{rad} U_{HR}$ where Z_{rad} denotes the acoustic radiation impedance composed of the radiation resistance and reactance, R_{rad} and X_{rad} respectively, i.e., $Z_{rad} = R_{rad} + jX_{rad}$. Incidentally for a circular pipe end of a baffled opening (of radius a, and for $ka \ll 1$), they are represented as

$$R_{rad} = \rho c k^2 (1 - k^2 a^2 / 6) / 2\pi, \; X_{rad} = 8\rho c k (1 - 4k^2 a^2 / 15) / 3\pi^2 a. \tag{2a, b}$$

Denoting Δl_{out} as the outer orifice-correction, and using the relationships

$$X_{\text{rad}} = \omega\rho\Delta l_{\text{out}} / S_{\text{A}}, \tag{3}$$

$Z_{\text{HR}} = Z_{\text{F}} + Z_{\text{rad}}$ and $R_{\text{HR}} = R_{\text{F}} + R_{\text{rad}}$, we have

$$Z_{\text{HR}} = R_{\text{HR}} + j\left\{(l_{\text{A}} + \Delta l_{\text{out}} + \Delta l_{\text{inn}})\omega\rho / S_{\text{A}} - \rho c^2 / \omega V_{\text{cav}}\right\}. \tag{4}$$

When an acoustic field containing a resonator has no frequency dependence except the resonator, the resonant frequency can be written as

$$f_{\text{res}} = (c/2\pi)\sqrt{S_{\text{A}} / V_{\text{cav}}(l_{\text{A}} + \Delta l_{\text{out}} + \Delta l_{\text{inn}})}. \tag{5}$$

For a resonator (with a circular aperture of radius a) embedded in a large plane wall and opening to a half space, the outer orifice-correction Δl_{out} is given as $(\Delta l_{\text{out}})_{\text{half space}} \approx 0.82a$. For this particular case, we write the resonant frequency f^*_{res} instead of f_{res}.

3. One-dimensional wave simulation

In the one-dimensional simulation by the plane wave model for a duct section with a Helmholtz resonator, ports (interfaces) 1, 2 and 3 are taken as shown in Fig. 1. For every port i, the sound pressure amplitude and the volume velocity amplitude p_i and U_i, respectively, are related by the acoustic impedance Z_i as $p_i = Z_i U_i$.

On the port 1, the sound pressure amplitude p_1, the volume velocity amplitude U_1, the incident wave pressure amplitude p_1^+, and the reflected wave pressure amplitude p_1^- are related as

$$p_1 = p_1^+ + p_1^-,\ Z_{\text{c}}U_1 = p_1^+ - p_1^-,\ Z_{\text{c}} = \rho c / S_{\text{D}} \tag{6a, b, c}$$

where Z_{c} denotes the characteristic acoustic impedance. These yield a relationship $2p_1^+ = p_1 + Z_{\text{c}}U_1$ and, taking $p_1 = Z_1U_1$ into consideration, we have

$$2p_1^+ = (Z_1 + Z_{\text{c}})U_1. \tag{7}$$

Between the ports 1 and 2, the pressure and volume-velocity amplitudes are related as

$$p_1 = p_2 = p_{\text{HR}} = Z_{\text{HR}}U_{\text{HR}},\ U_1 = U_{\text{HR}} + U_2 \tag{8a, b}$$

and, taking $p_1 = Z_1U_1$ and $p_2 = Z_2U_2$ into account, we have

$$Z_1U_1 = Z_2U_2 = Z_{\text{HR}}U_{\text{HR}} \tag{9}$$

For a two-port element 2-3 between the ports 2 and 3, the pressure and volume-velocity amplitudes are related in terms of the fundamental matrix, for instance, as

$$p_2 = Ap_3 + BU_3,\ U_2 = Cp_3 + DU_3 \tag{10a, b}$$

where A, B, C and D are the four terminal constants. In case of a straight duct of length l, for instance, they are represented as $A = D = \cos kl$, $B = jZ_{\text{c}}\sin kl$, $C = jZ_{\text{c}}^{-1}\sin kl$.

Taking $p_3 = Z_3U_3$ into account, Eq. (8) and (10) yield

$$1/Z_1 = 1/Z_{\text{HR}} + 1/Z_2,\ Z_2 = (AZ_3 + B)/(CZ_3 + D). \tag{11a, b}$$

4. Dissipation and Transmission factors

The dissipation factor of a Helmholtz resonator is defined as the ratio of the dissipated sound power $P_{\mathrm{HR}} = R_{\mathrm{HR}} |U_{\mathrm{HR}}|^2 / 2$ to the incident sound power $P_1^+ = |p_1^+|^2 / 2Z_{\mathrm{c}}$, i.e.,

$$\delta_{\mathrm{HR}} = Z_{\mathrm{c}} R_{\mathrm{HR}} |U_{\mathrm{HR}} / p_1^+|^2 . \tag{12}$$

For a duct section between the ports 1 and i, the transmission factor is defined as the ratio of the transmitted sound power $P_i = R_i |U_i|^2 / 2$ to the incident sound power, i.e.,

$$\tau_{i1} = Z_{\mathrm{c}} R_i |U_i / p_1^+|^2 . \tag{13}$$

Eq. (11) can be applied to general tree-type-network ducts. As a special case for a N resonator chain, as seen in Fig. 3, made up with N sections which each are similar to the element 1-3, and are numbered $n = 1, 2, \cdots, N$ with the ports $2n-1$ through $2n+1$, we apply Eq. (11) in the form

$$1/Z_{2n-1} = 1/Z_{\mathrm{HR}}^{(n)} + 1/Z_{2n+1}, \quad Z_{2n} = (A^{(n)} Z_{2n+1} + B^{(n)}) / (C^{(n)} Z_{2n+1} + D^{(n)}) \tag{14a, b}$$

where the superscript (n) denoted the n th resonator or the n th duct section. When the termination impedance Z_{2N+1} of N th (the last) resonator is given, the impedances Z_{2n} and Z_{2n-1} of the duct section for n th resonator can be determined by using Eq. (14b) for $n = N, N-1, \cdots, 2, 1$ in tern. For a N resonator chain between the ports 1 and $2N+1$, the dissipation factor of i th resonator δ_i, and the transmission factor between the ports 1 and $2i+1$, $\tau_{2i+1,1}$ are written as

$$\delta_i = Z_{\mathrm{c}} R_{\mathrm{HR}}^{(i)} |U_{\mathrm{HR}}^{(i)} / p_1^+|^2 = 4 Z_{\mathrm{c}} R_{\mathrm{HR}}^{(i)} |U_1 / 2p_1^+|^2 |U_{\mathrm{HR}}^{(i)} / U_{2i-1}|^2 \prod_{n=1}^{i-1} |U_{2n+1} / U_{2n-1}|^2 ,$$

$$\tau_{2i+1,1} = Z_{\mathrm{c}} R_{2i+1} |U_{2i+1} / p_1^+|^2 = 4 Z_{\mathrm{c}} R_{2i+1} |U_1 / 2p_1^+|^2 \prod_{n=1}^{i} |U_{2n+1} / U_{2n-1}|^2 . \tag{15a, b}$$

These can be determined by using Eq. (7), i.e., $2p_1^+ / U_1 = Z_{\mathrm{c}} + Z_1$ and Eq. (14), and the following relationships derived from Eqs. (9) and (10b);

$$U_{\mathrm{HR}}^{(i)} / U_{2i-1} = Z_{\mathrm{HR}}^{(i)} / Z_{2i-1} \text{ and } U_{2n-1} / U_{2n+1} = (Z_{\mathrm{HR}}^{(n)} + Z_{2n})(C^{(n)} Z_{2n+1} + D^{(n)}) / Z_{\mathrm{HR}}^{(n)} . \tag{16a, b}$$

The acoustic impedance Z_{HR} of a resonator is identical to the combined impedance of M resonators of the acoustic impedance MZ_{HR} attached along the same circumferential line of the duct wall. This implies that the resonator chain corresponds substantially to the sidewall resonators of $M \times N$ individual resonators.

5. Outer orifice Corrections

To determine the mass end correction Δl_{out} for a resonator attached to the sidewall of a circular duct ($\theta_{\mathrm{A}} = 90°$), a series of full wave simulations by the 3-D BEM (the 3-dimensional boundary element method [2]) was conducted. As shown in Fig. 2, two methods, i.e., Eq. (5) and Eq. (4) were used to determine Δl_{out}; f_{res} for Eq. (5) and X_{rad} for Eq. (4), respectively, were given by the BE simulation. In case of a resonator attached to the plate of a duct end ($\theta_{\mathrm{A}} = 0°$), Δl_{out}

determined by f_{res} agrees excellently, while that by X_{rad} has some discrepancy, compared to the Rschevkin's expression. The correction Δl_{out} for $\theta_A = 90°$ is far greater than that for $\theta_A = 0°$.

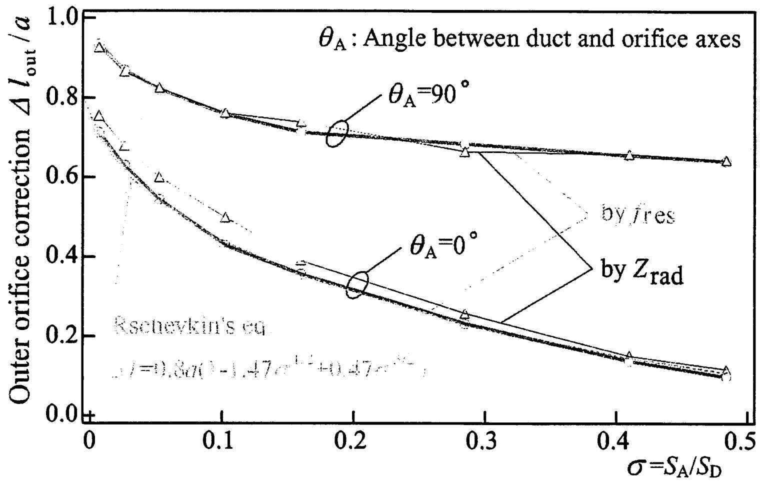

Figure 2 Dependence of outer orifice correction Δl_{out} on aperture directions relative to duct axis.

6. Application of End-Correction for sidewall resonators

To confirm the effectiveness of the correction Δl_{out} for $\theta_A = 90°$, several simulations are carried out. Fig. 3 shows the dissipation and transmission factors of a duct section with a resonator array (when the total number of resonator is 5, and for the section between the ports 1 and 11). Here δ_{1-11} was given as the difference between the absorption factor and the transmission factor.

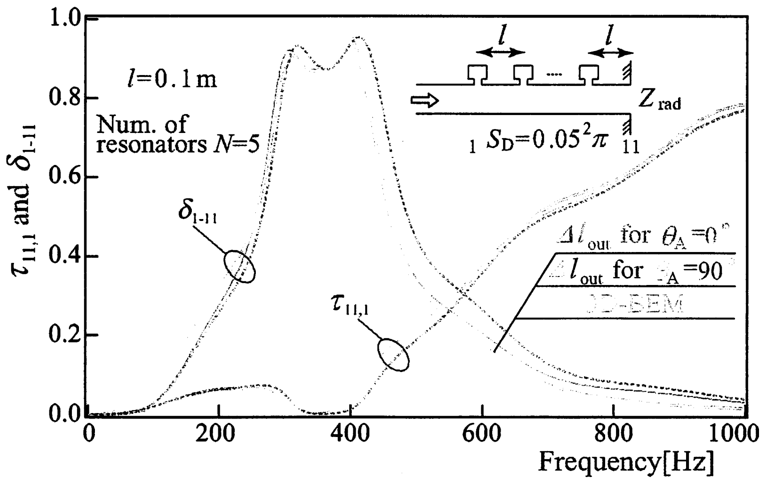

Figure 3 Comparison of 1-D and 3-D model in terms of sound dissipation factors δ_{1-11} and transmission factors $\tau_{11,1}$, for a duct section containing sidewall resonators.

When the correction Δl_{out} for $\theta_A = 90°$ is used, the results of $\delta_{1\text{-}11}$ and $\tau_{11,1}$, respectively, by 1-D model approaches to that by 3-D BEM. Fig. 4 shows the dissipation factor of each resonator given by Eq. (15a) for the same duct section as shown in Fig. 3. They each play different roles in the dissipation of the sound.

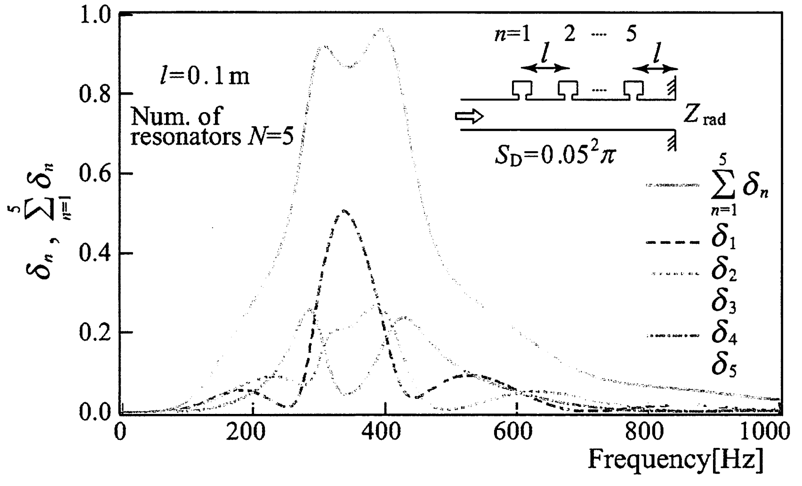

Figure 4 Each resonator contribution to sound dissipation (1D model).

7. Conclusion

It is found that, for a given ratio of the resonator aperture area and the duct cross-sectional area, the outer orifice correction Δl_{out} for resonators attached to the sidewall of ducts is far larger than that attached to the end plates of ducts. Use of the correction Δl_{out} for side-branch resonators improves remarkably the prediction accuracy of the one-dimensional wave simulation for duct sections which have sidewall resonators.

References

1. L. Cremer and H. A. Muller, Principles and Applications of Room Acoustics, Applied Science Publishers, Chapter IV.9, 1982.
2. M. Terao and H. Sekine, On Substructure Boundary Element Techniques to Analyze Acoustic Properties of Air-duct Components, Proc. Inter-noise 87, 1523-1526, 1987.