

# ON CHARACTERISTIC TRANSMISSION AND REFLECTION COEFFICIENTS OF SPLITTER DUCT ATTENUATORS

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## 1. INTRODUCTION

The attenuation of a splitter duct silencer has been estimated by applying such theories as P.M. Morse (JASA, 1939), R.A.Scott [1] and U.A.Kurze (JS&V, 1972), or by extrapolating such an empirical chart of A.J.King (JASA, 1958). These are basically appropriate only for the fundamental wave mode in the straight channel of an attenuator of infinite length. However higher order mode waves are also important because this type of silencers are employed in ducts of large width. To realize acoustical estimation of a finite length attenuator including higher order modes, we investigated a determination method of the characteristic transmission and reflection coefficients of an attenuator by employing the superposition principle of traveling waves, and carried out tests by BE numerical simulations and by an experiment on a two-dimensional attenuator with two air-channels for the two typical configurations; i.e., installation in a straight duct and installation between a couple of right-angle elbows, at frequencies below second order mode cut-on in the far field.

## 2. DECOMPOSITION OF TRAVELING WAVE PRESSURES

Taking an interface, coordinates and the origin (0,0) as shown in Figure 1 (a) in the straight duct region of each duct connected to the attenuator where evanescent modes from the discontinuities have died out, the sound pressure  $p(x, y)$  is represented as

$$p(x, y) = \sum_{n=0}^N a_{(n)} \exp(-jk_x^{(n)}x) \cos(k_y^{(n)}y) + \sum_{n=0}^N b_{(n)} \exp(+jk_x^{(n)}x) \cos(k_y^{(n)}y) \quad (1)$$

where  $n(=0,1,2,\dots,N)$  represents the number of the pressure nodes in the  $y$  direction, here  $N$  is the highest number of the propagating modes for the

frequency of interest in the straight duct. The quantities  $k_y^{(n)} = n\pi/L_y$  and  $k_x^{(n)} = \{(\omega/c)^2 - (k_y^{(n)})^2\}^{1/2}$  are the wave numbers of the  $n$ th mode in the  $y$  and  $x$  directions, respectively. The quantities  $a_{(n)}$  and  $b_{(n)}$  denote the outgoing and incoming plane wave pressures, respectively, of the  $n$ th mode at the origin. These plane wave pressures can be determined by solving simultaneously a set of equations (1) corresponding to the sound pressures  $p(x, y)$  at  $2(N+1)$  positions. These sound pressures were observed by conducting a BE numerical simulation [2] for both configurations of Figure 1 (a) and (b), and by an experiment for the configuration of Figure 1 (a).

### 3. DETERMINATION OF CHARACTERISTIC COEFFICIENTS

The outgoing wave pressure,  $a_{\ell(n)}$ , of the  $n$ th mode on a interface  $\ell$  is represented by the superposition of the contributions of the incoming waves,  $b_{\ell'(n')}$ , of all of the modes  $n'$  of all of the interfaces  $\ell'$  as

$$a_{\ell(n)} = \sum_{\ell'=1}^L \sum_{n'=0}^{N_{\ell'}} \tau_{\ell(n) \ell'(n')} b_{\ell'(n')}, \quad \text{for } \ell = I, II, \dots, L, \text{ and } n = 0, 1, 2, \dots, N_{\ell} \quad (2)$$

where  $N_{\ell}$  and  $N_{\ell'}$  are the highest numbers of the propagating modes in the  $\ell$ th and  $\ell'$ th straight duct sections, respectively,  $L$  denotes the number of the ducts connected to the attenuator, and  $\tau_{\ell(n) \ell'(n')}$  represents the characteristic transmission (or reflection when  $\ell = \ell'$ ) coefficient between an

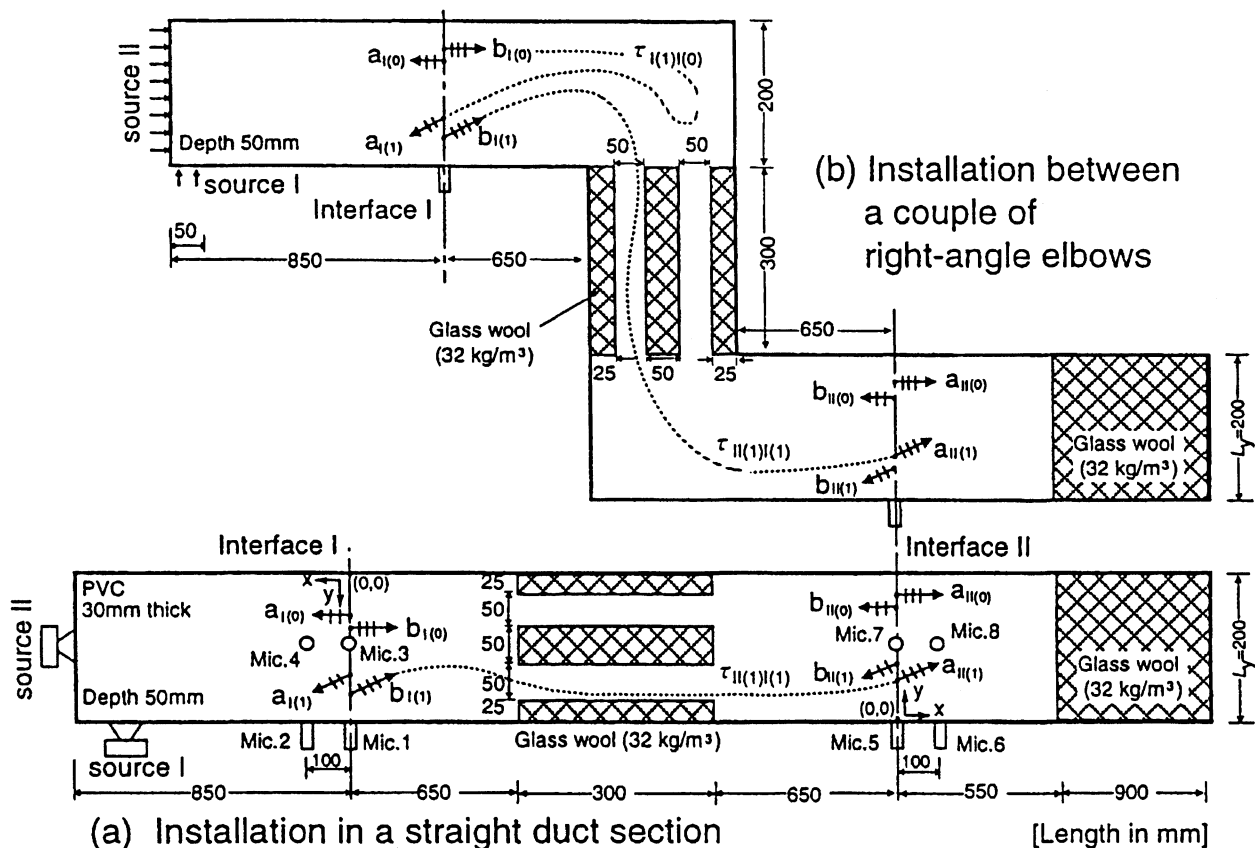
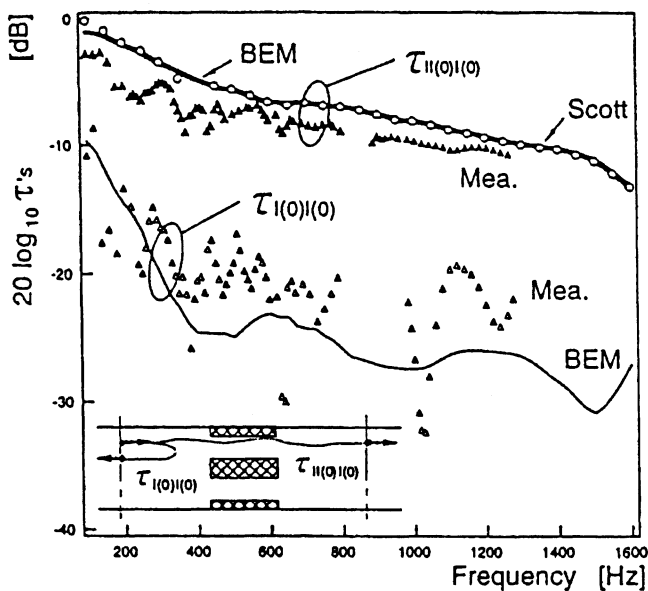
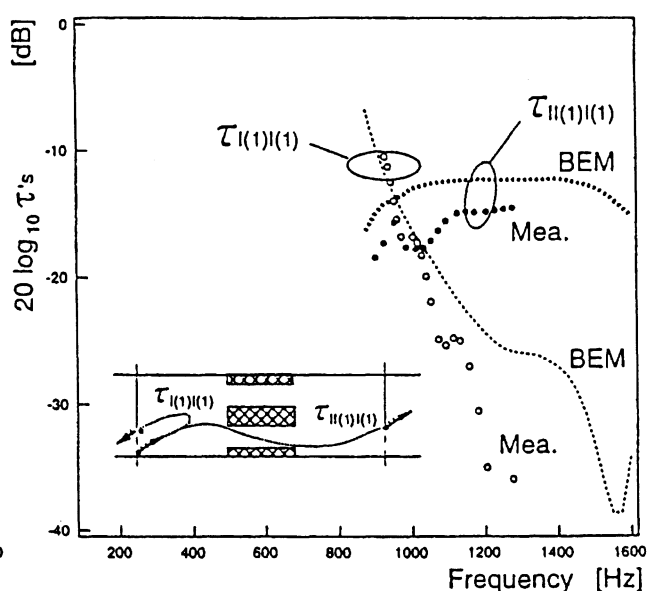


Figure 1 Configurations of a splitter duct attenuator and test setup

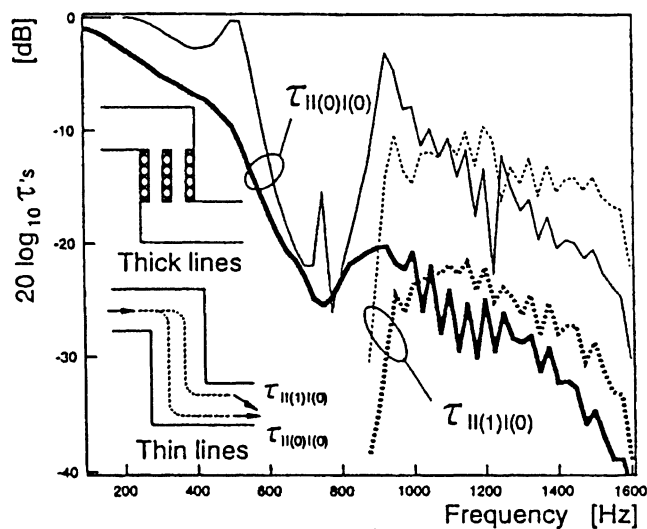


(a) Fundamental mode incidence

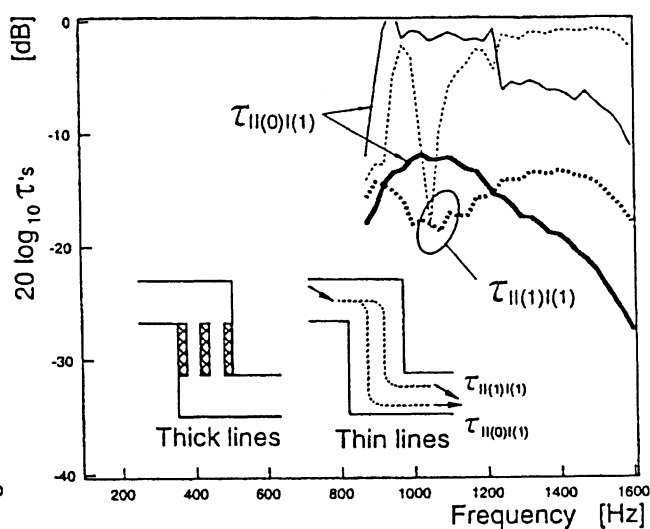


(b) 1st order mode incidence

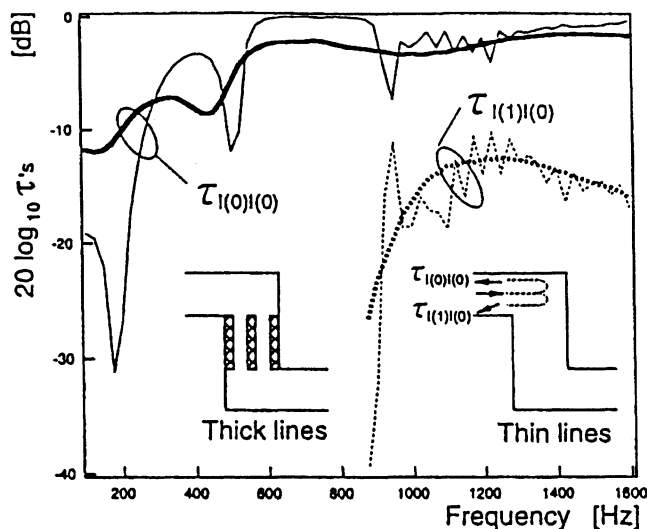
Figure 2 Transmission and reflection factors for installation in a straight duct



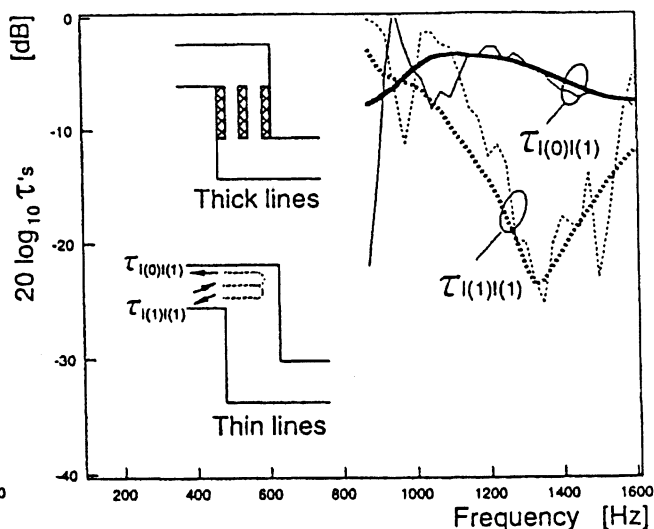
(a) Transmission factors for fundamental mode incidence



(c) Transmission factors for 1st order mode incidence



(b) Reflection factors for fundamental mode incidence



(d) Reflection factors for 1st order mode incidence

Figure 3 Transmission and reflection factors for installation between two elbows

incoming wave  $b_{\ell'(n')}$  and its contribution to an outgoing wave  $a_{\ell(n)}$ . To determine these coefficients,  $M$  different cases of the sound fields were generated and measured, where  $M = N_I + N_{II} + \dots + N_L$ . Having and solving a set of  $M$  independent equations (2) for every  $\ell(n)$ , we can determine the unknowns  $\tau_{\ell(n) \ell'(n')}$  of total  $M$  without using anechoic terminations.

## 4. TEST RESULTS

### Installation in a Straight Duct

Figure 2 shows the transmission and reflection coefficients of the attenuator installed in a straight duct section as shown in Figure 1(a). The glass fiber blanket of  $32 \text{ kg/m}^3$  was used in its smaller direction of flow resistance, and its acoustic properties for the numerical simulation and the Scott theory were measured by an impedance tube method. Disagreement between the coefficients by the measurement (Mea.) and the BE simulation (BEM) may partly be caused by discrepancy in the blanket acoustic properties between the measurement and the numerical simulation. For fundamental mode incidence at this configuration, the reflection coefficients,  $\tau_{I(0) I(0)}$  and  $\tau_{I(1) I(0)}$ , are rather small and the transmission coefficient corresponding to the attenuation by Scott theory has good agreement with that ( $\tau_{I(0) I(0)}$ ) by the numerical estimation. It should be noted that the transmission coefficient between the first order mode,  $\tau_{II(1) I(1)}$ , is not small enough to be disregarded.

### Installation between a Couple of Right-Angle Elbows

Figure 3 shows the transmission and reflection coefficients of the attenuator between a couple of right-angle elbows as shown in figure 1(b). Compared to the installation in a straight duct, every corresponding reflection coefficient becomes larger and the transmission coefficients between the fundamental modes,  $\tau_{II(0) I(0)}$ , becomes far smaller. Compared with this, those of cross mode incidence,  $\tau_{II(1) I(1)}$  and  $\tau_{II(0) I(1)}$ , become important.

## 5. CONCLUSIONS

The transmission and reflection coefficients of a splitter duct attenuator in two typical configurations have been obtained for the fundamental and the first order modes. For the transmission coefficient between the fundamental modes, the attenuation by Scott theory of a lined duct channel of infinite length can be used as far as an attenuator is installed in a straight duct section. But this does not hold when it is installed between a couple of right-angle elbows. The transmission coefficients related to the first order modes cannot be disregarded compared with those related to fundamental modes.

## REFERENCES

1. R.A Scott, *Proceedings of the Physical Society* 58, 358-368(1946).
2. M.Terao and H.Sekine, "A numerical analysis of sound field of a long space by a sub-region coupling approach," *Proceedings of Internoise 96*, Liverpool, England, 3007-3010, 1996.