Calculation of Foster-type Equivalent Network for Stripline 45 Degrees Bend Based on Novel Calculation Method of the Eigenmode

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***Abstract* — Foster-type equivalent network for stripline 45 degrees bend of equal or different stripline width is derived based on eigenmode method. The key step is how to calculate eigenmodes of the planar junction of 45° bend up to necessary mode number with exactness. We propose a new idea of three stage cascade-connected stripline configuration and double symmetry with magnetic wall, which is practically carried out with success. The derived equivalent network is applied to the calculation of frequency characteristics and field distribution under the operating.**

***Index Terms* — Stripline 45 degrees bend, 3-stage stripline circuit, double symmetry with magnetic wall, Foster-type equivalent network, T-type equivalent network**

I. INTRODUCTION

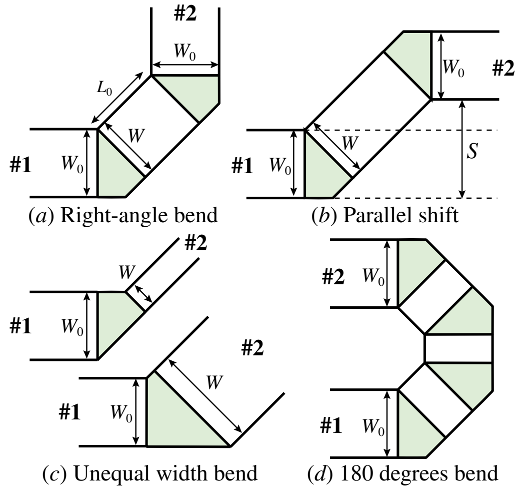
Stripline 45 degrees bend with input/output stripline of equal width or different width as shown in Fig.2(*a*)(*b*) is one of basic components at MIC, whose application examples are shown in Fig.1. So far, the element parameters of equivalent network for stripline bend have been given approximately or empirically, but can not be used for exact analysis and design of stripline circuit. Hence, rigorous equivalent network for stripline 45 degrees bend is strongly needed, which is given by eigenmode expansion method explained in section II. The key step is how to calculate the eigenmode of the planar junction defined in Table 2. We propose a new idea to calculate that, i.e. 3-stage cascade-connected stripline configuration and double symmetry with magnetic wall. The detail of our method and calculated results are explained in section III. Once the eigenmodes are calculated by this way, the eigenmodes for any width ratio can be obtained by using the similarity principle of 45 degrees bend.

II. EQUIVALENT NETWORK BY MODE THEORY

Stripline 45 degrees bend shown in Fig.2(*a*)(*b*) can be modelled to planar circuit with magnetic side wall, which is divided into two planar waveguides and one planar junction. Equivalent network for waveguide is given by multi-transmission line and that for planar junction by Foster-type equivalent network derived from eigenmode expansion method. Therefore whole equivalent network is given by Fig.2(*c*) or vector notation in Fig.2(*d*), whose network parameters are given in Table 1, after the calculation of eigenmode of 45 degrees bend defined in Table 2. How to calculate the eigenmode is explained in the following section.

III. CALCULATION OF THE EIGENMODE FOR PLANAR JUNCTION

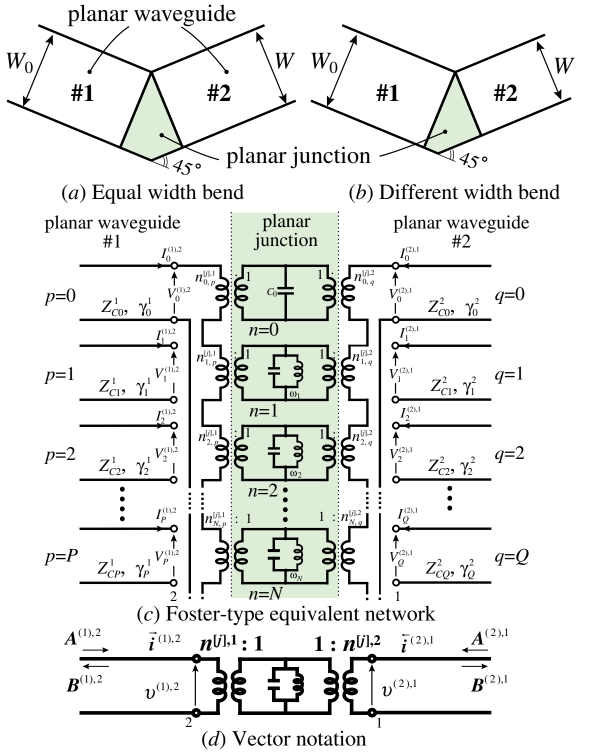
In order to calculate 2D eigenmode for planar junction of stripline 45 degrees bend with equal or different stripline width shown in Fig.2, we propose to use 3-stage stripline



(*a*) Right-angle bend (*b*) Parallel shift

(*c*) Unequal width bend (*d*) 180 degrees bend

Fig.1 Application of 45° bend to various stripline circuits



(*a*) Equal width bend (*b*) Different width bend

(*c*) Foster-type equivalent network

(*d*) Vector notation

Fig.2 45 degrees bend and its equivalent network

Table 1 Network parameters for equivalent network

$$c^i_p(s^i) = \sqrt{\varepsilon_p}\cos\frac{p\pi}{W^i}s^i \qquad \left(p = 0,1,2,\cdots,P\right)$$

$$\gamma^i_p = \sqrt{\left(p\pi/W^i\right)^2 - \omega^2\varepsilon\mu}\ , \quad Z^i_{C\,p} = \frac{j\omega\mu}{\gamma^i_p}\frac{d}{W^i} \tag{1}$$

$$C_0 = \varepsilon\frac{S}{d}\ [\mathrm{F}] \quad L_n = \frac{\varepsilon\mu}{k_n^2}\frac{1}{C_0}\ [\mathrm{H}] \quad \omega_n = \frac{k_n}{\sqrt{\varepsilon\mu}}$$

$$n^{\ i}_{n\,p} = \frac{1}{W^i}\int_0^{W^i}\varphi_n(x,y)c^i_p(s^i)\,ds \qquad \boldsymbol{n}^{[j],i} = \left[n^{\ i}_{n\,p}\right] \tag{2}$$

and double symmetry with magnetic wall. Four planar junctions of stripline 45 degrees bend are connected in back to back as shown in Fig.3(*a*), then whole structure can be understood as 3 striplines are connected in cascade as shown in Fig.3(*b*), whose equivalent network is given in Fig.3(*c*) or (*d*) in vector notation consisting of three multi-transmission lines and two multi-port ideal transformers. Mode admittance matrix looking toward leftside at port 2 of #3 waveguide $\overleftarrow{\boldsymbol{Y}}_{in}^{(3),2}$ can be derived based on definition of port mode voltage/current column matrix and their relation given by network theory. Therefore, eigenvalue equation become

$$\overleftarrow{\boldsymbol{i}}^{(3),2} = \overleftarrow{\boldsymbol{Y}}_{in}^{(3),2} \cdot \boldsymbol{\upsilon}^{(3),2} = \mathbf{0}.$$

Eigenmodes for 3-stage configuration can be classified into 4 types of mode distribution because of double symmetry explained in Fig.4. Eigenmodes for 45° bend are obtained by selecting mode distribution which satisfies magnetic wall symmetry about *A-A*', *B-B*'. For example, calculated eigenvalues up to 20th and eigenmodes up to 9th for three-stage configuration with equal stripline width(R=1.0) are shown in Table 3 and Fig.4. We can select three modes (n=3,6,9) from Fig.5, which have double magnetic wall symmetry.

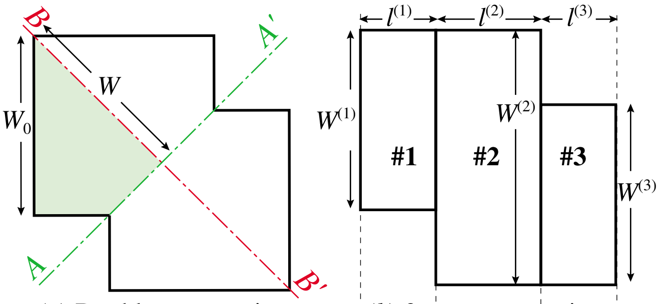
Table 2 Eigenfunction of planar junction

$$\frac{\partial^2\varphi_n}{\partial x^2} + \frac{\partial^2\varphi_n}{\partial y^2} + k_n^2\varphi_n(x,y) = 0 \qquad (3)$$

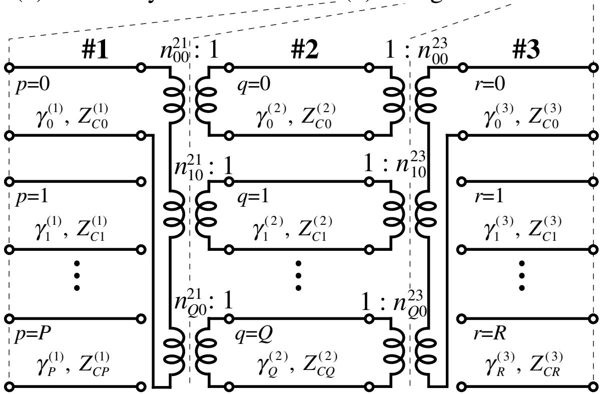
$$\text{where } \nabla_t\varphi_n \cdot \boldsymbol{n} = \frac{\partial\varphi_n}{\partial n} = 0 \quad \text{(Magnetic wall)} \qquad (4)$$

$$k_0 = 0 \ \le\ k_1 \ \le\ k_2 \ \le \cdots \quad \text{(Eigenvalue)}$$

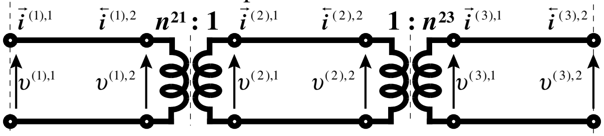
$$\frac{1}{S}\iint_S \varphi_n(x,y)\cdot\varphi_m(x,y)\,dxdy = \delta_{nm} \qquad (5)$$



(*a*) Double symmetries (*b*) 3 stage connection



(*c*) Equivalent network by multi-transmission line and multi-port ideal transformer



(*d*) Vector representation

Fig.3 Stripline 3-stage cascade-connected configuration and its equivalent network

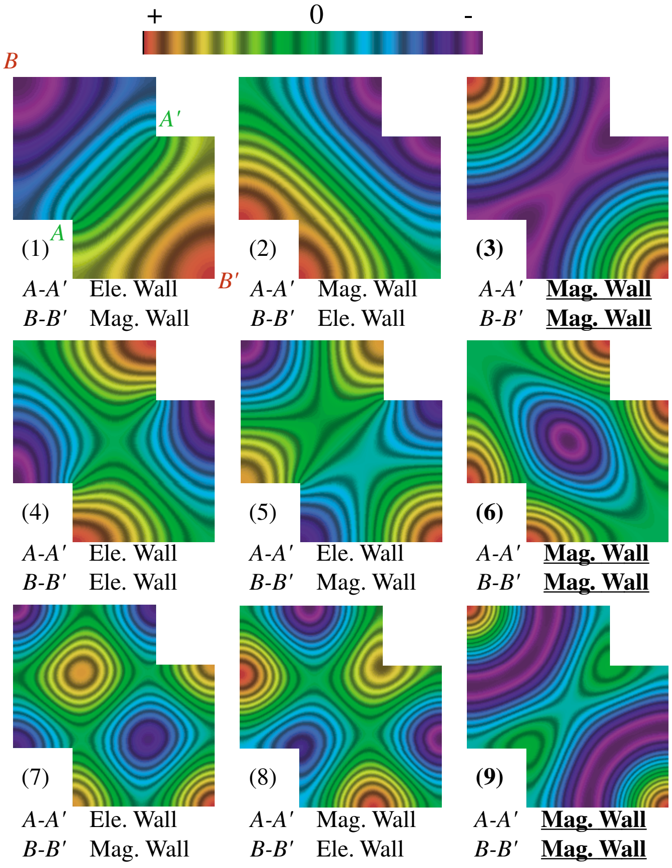


Fig.4 Calculated voltage distribution of eigenmode for 3-stage cascade-connected circuit

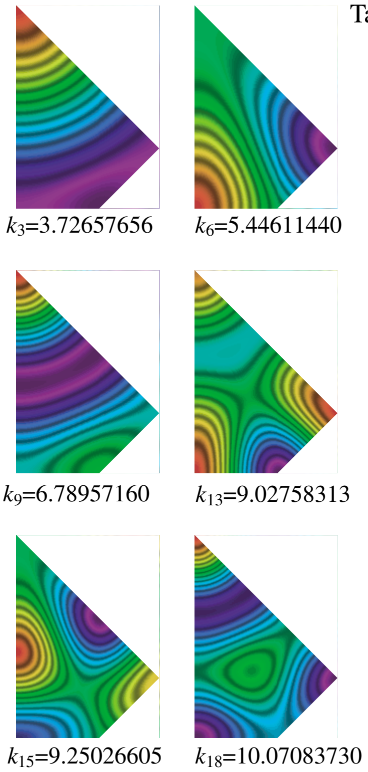


Fig.5 Desired voltage distribution of eigenmode for 45 degrees bend

Table 3 Eigenvalue of planar junction (width ratio R=1.0)

(1)	1.971518821
(2)	2.957999453
(3)	**3.726576563**
(4)	3.726807703
(5)	4.42951562
(6)	**5.446114404**
(7)	6.444558016
(8)	6.450420734
(9)	**6.789571595**
10	6.78967534
11	7.338356578
12	7.96326174
13	**9.027583132**
14	9.027715263
15	**9.250266053**
16	9.65710564
17	9.668900991
18	**10.0708373**
19	10.07114474
20	10.62592328

By this way, we can select mode distribution and corresponding eigenvalues, whose results are summarized in Table 3 by bold number and in Fig.5 for 45° bend (higher modes are added in Fig.5). In this way eigenvalues k_nW up to 30 are calculated as a function of width ratio $\boldsymbol{R=W/W_0}$ and shown in Fig.6. Eigenvalues in Fig.6 can be calculated only $1/\sqrt{2} < R < \sqrt{2}$, but the other ranges $R < 1/\sqrt{2}$ (narrow output line) or $R > \sqrt{2}$ (wide output line) can be derived by using the similarity principle in Fig.7. Therefore, the eigenvalue ranged from R=0.0 to ∞ can be obtained but only R=0.0~3.0 are shown in Fig.8.

IV. FREQUENCY CHARACTERISTICS

The calculated eigenmode of stripline 45° bend gives the network parameter of Foster-type equivalent network by Table 1. Then, mode impedance is given by following equation and frequency characteristics is shown in Fig.9.

$$Z^{i,j}_{p,q} = -j\frac{1}{C_0}\sum_{n=0}^{N}\frac{\omega}{\omega^2-\omega_n^2}\, n^i_{n,p}\, n^j_{n,q}$$

It turns out that complete transmission for equal input/out stripline width(R=1.0) and incomplete transmission for unequal width(R≠1.0) at low frequency because of reflection due to unequal width. Also, less width ratio R, wider bandwidth, because of small planar junction. Phase delay φ_{21} and effective length of 45 degrees bend are calculated and

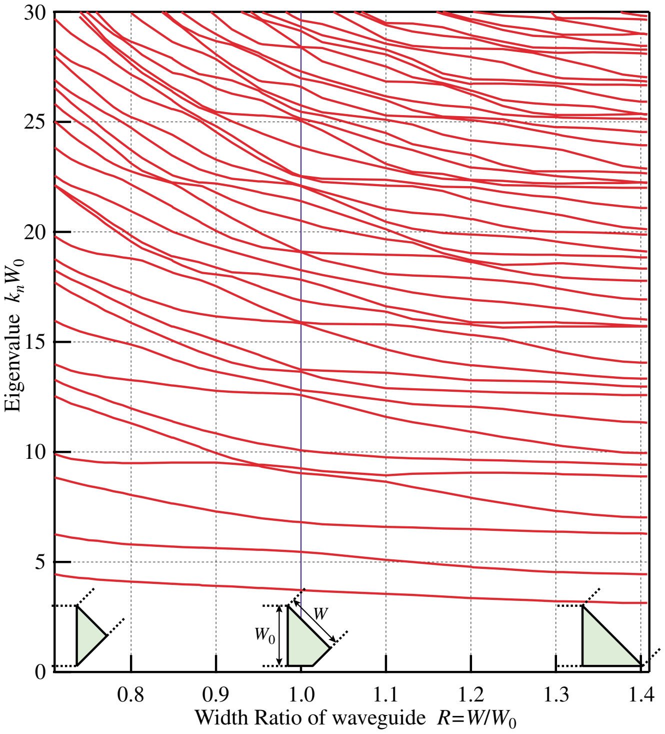


Fig.6 Calculated eigenvalue k_n of planar junction (45 degrees bend) $\left(1/\sqrt{2} < R < \sqrt{2}\right)$

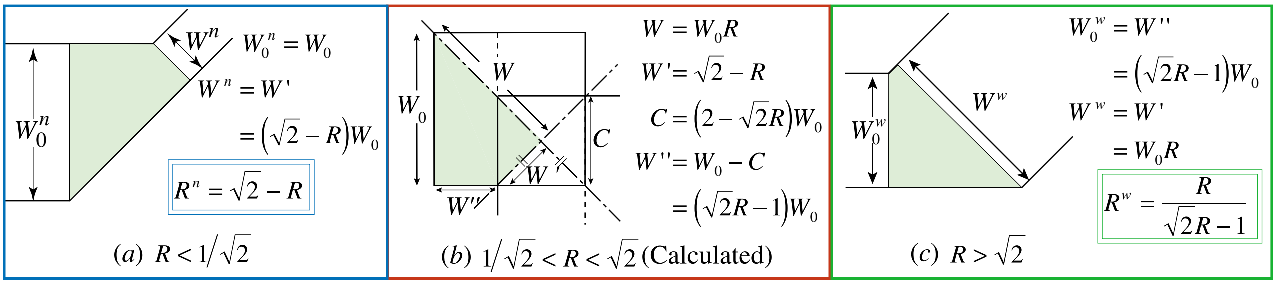


Fig.7 Derivation of the eigenmodes for any width ratio using similarity principle

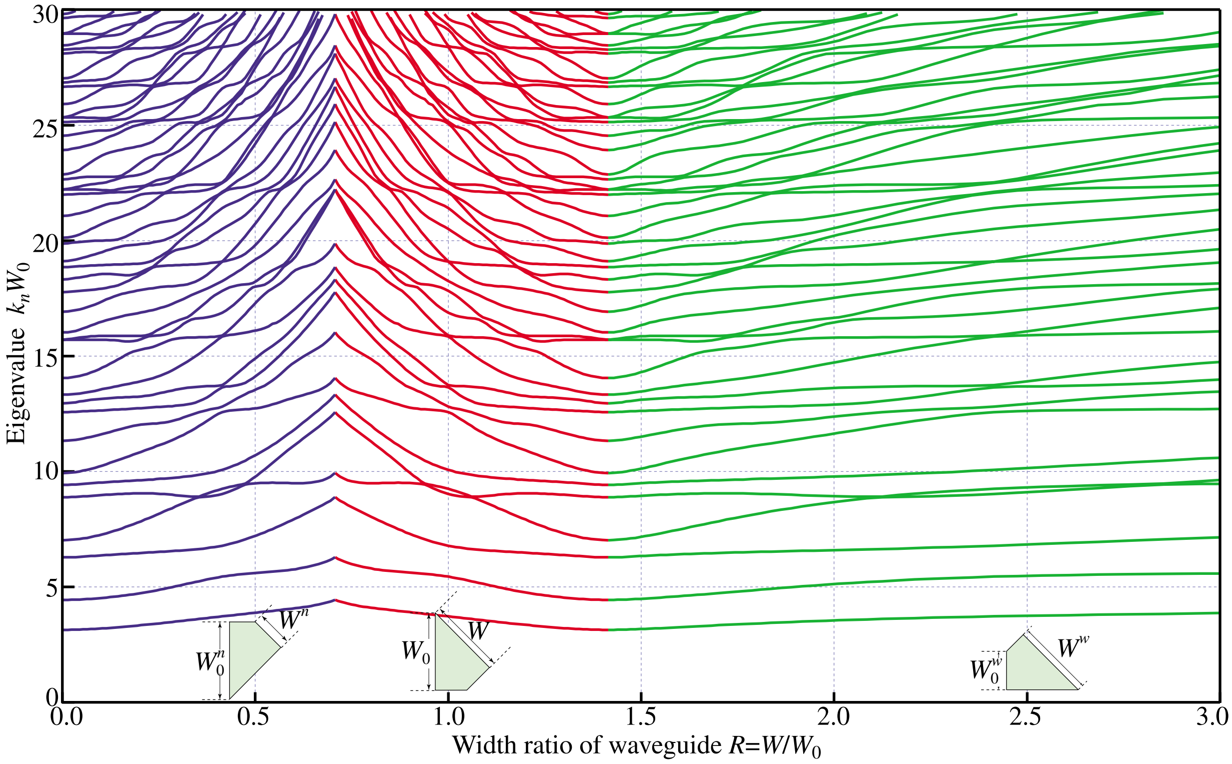


Fig.8 Eigenvalue for various width ratio R

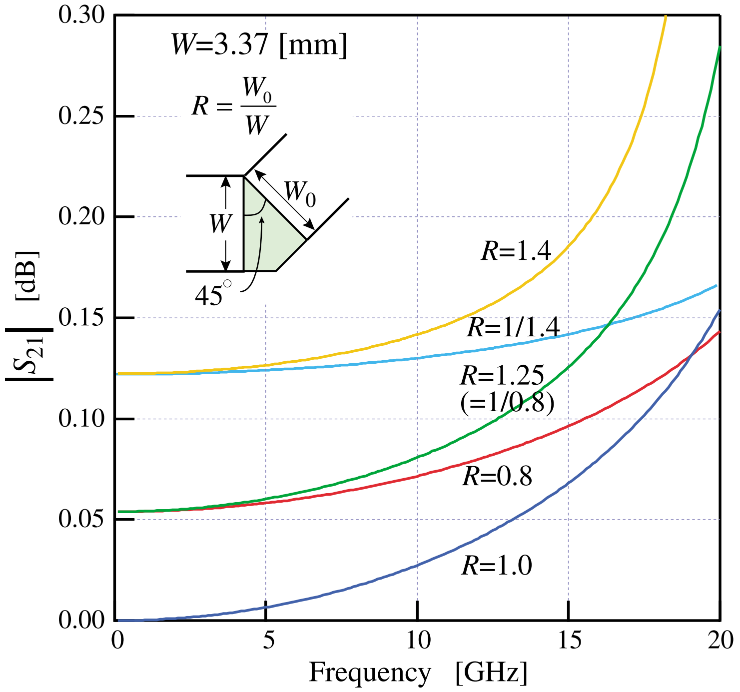


Fig.9 Frequency characteristics for 45 degrees bend with equal width (R=1.0) and different width ($R=W_0/W$)

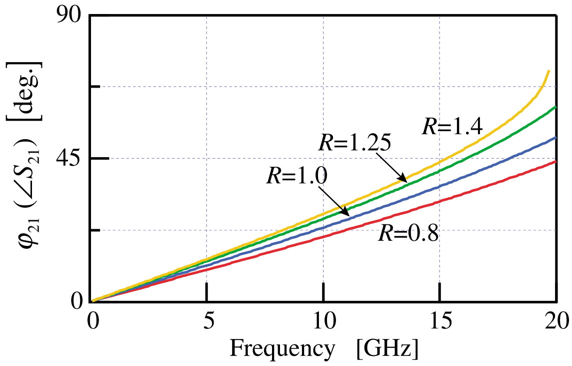


Fig.10(a) Phase delay φ_{21} ($\angle S_{21}$) of 45° bend for various R

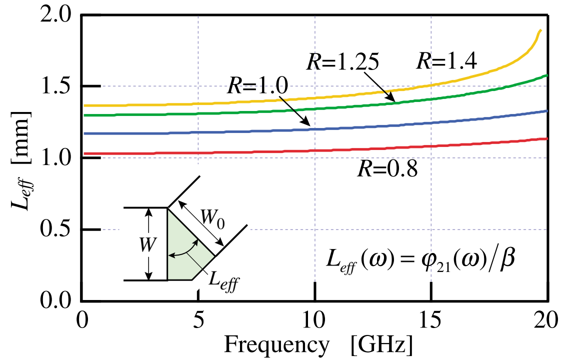


Fig.10(b) Effective length L_{eff}

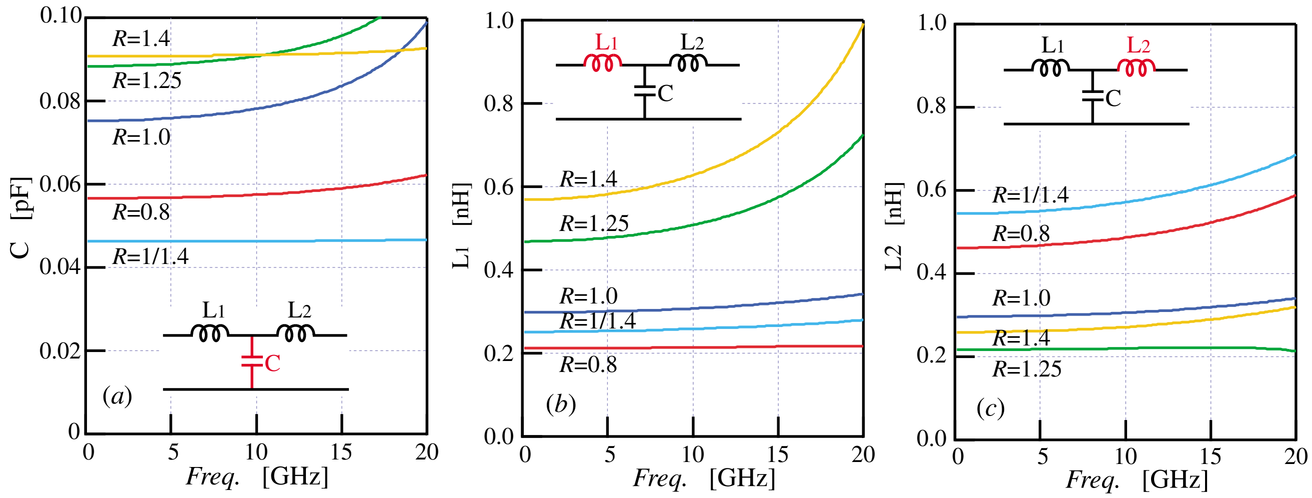


Fig.11 Network parameter of T-type equivalent network (a) Capacitance (b)(c) Inductance

Table 4 Comparison of capacitance

R	C [pF]	C' [pF]
1.25	0.0883956	0.0883958
1.0	0.0752587	0.0752590
0.8	0.0565732	0.0565733
1/1.4	0.0463403	0.0463403

shown in Fig.10(a) and (b), respectively.

V. T-TYPE EQUIVALENT NETWORK

45 degrees bend can be given by T-type equivalent network shown in Fig.11. Network parameter of T-type equivalent network (C, L1, L2) are calculated and shown in Fig.11, which is derived from mode impedance for Foster-type equivalent network and effective dominant impedance matrix for LC circuit. L1 and L2 become equal at R=1.0 and thier difference increase with increasing width ratio. Table 4 compared with C of T-type equivalent network and capacitance $C' = \varepsilon_0 \varepsilon_S S/d$ of planar junction of 45 degrees bend around DC, and agreed well.

VI. CONCLUSION

Novel calculation method of eigenmode for stripline 45 degrees bend is proposed and practically carried out, which can give Foster-type equivalent network of stripline 45 degrees bend. Wideband frequency characteristics of 45 degrees bend are calculated for various width ratio R, which can lead to the discussion of the optimum dimension.

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