Analysis of stripline right-angle bend with slant-wise corner cut based on eigenmode expansion method and Foster-type equivalent network

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Abstract Stripline right-angle bend with slant-wise corner cut is analyzed based on planar circuit modelling, mode theory and thus derived equivalent network. Wide-band frequency characteristics are calculated with corner cut as a parameter, which leads to the discussion about transmission vs bandwidth and exact determination of the optimum cut. These results are compared with that of square shaped corner cut already presented.

1. Introduction

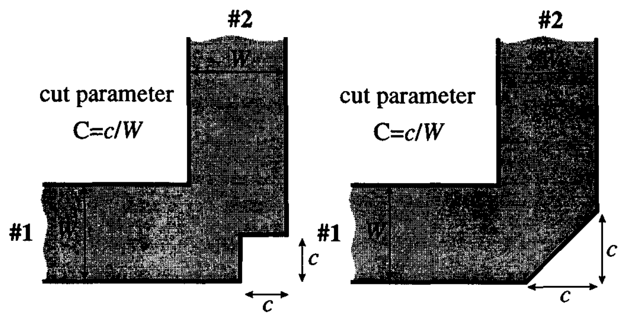
Stripline right-angle bend with slant-wise corner cut shown in Fig.1(b) is analyzed based on following systematic way and equivalent network, succeeding the analysis of square-shaped corner cut case (Fig.1(a)).

1. Stripline discontinuity is modelled to 2-dimensional planar circuit with magnetic side wall.
2. Then, 2D planar circuit is divided into planar waveguide section and planar junction section.
3. Applying eigenmode expansion method for each section, equivalent multi-transmission line network is derived for waveguide section and Foster-type equivalent network for junction section, respectively.

In our analysis, frequency characteristic are calculated with cut C as a parameter, and the optimum performance is discussed in terms of transmission vs bandwidth relation. Finally, it is shown that slantwise corner cut realizes better performance compared with square-shaped corner cut. Also field distribution at operation are now under calculation.

2. Derivation of equivalent network for stripline discontinuity based on mode theory

2-dimensionally modelled stripline circuit can be divided into planar waveguide section (ℓ^i-s^i local coordinate system, i=1,2) and planar junction section (x-y coordinate system) as shown in Fig.2(a). Equivalent net-



(a) square-shaped corner cut (b) slant-wise corner cut

Fig.1 Typical example of right-angle bend with corner cut

work for each section is derived based on mode theory.

2-1. Equivalent network for planar waveguide

When the width mode function for i-th planar waveguide in Fig.2(a) is defined by eq.(1), p-th mode voltage V_p^i and mode current I_p^i can be properly defined, and related by conventional transmission line equations.

$$f_p^i(s^i) = \sqrt{\varepsilon_p}\cos\frac{p\pi}{W^i}s^i \qquad (1)$$

$$\varepsilon_p = 1 \quad (p=0),\quad 2 \quad (p \ge 1) \quad (p = 0,1,2,\cdots)$$

Therefore, the equivalent multi-transmission line network is shown in Fig.2(b), where mode propagation constant γ_p^i and characteristic mode impedance $Z_{C_p}^i$ are also given by following equations.

$$\gamma_p^i = \sqrt{(p\pi/W^i) - \omega^2\varepsilon\mu}\ ,\quad Z_{C_p}^i = \frac{j\omega\mu}{\gamma_p^i}\frac{d}{W^i}\quad [\Omega] \qquad (2)$$

In order to simplify multi-transmission line network representation, vector notation is introduced as shown in Fig.2(c) ,where v^i and i^i are mode voltage and current column matrix, γ^i and Z_C^i are propagation constant and characteristics mode impedance in diagonal matrix.

2-2. Equivalent network for planar junction

When the eigenmode function defined in Table.1 are once calculated for any planar junction S, the Foster-

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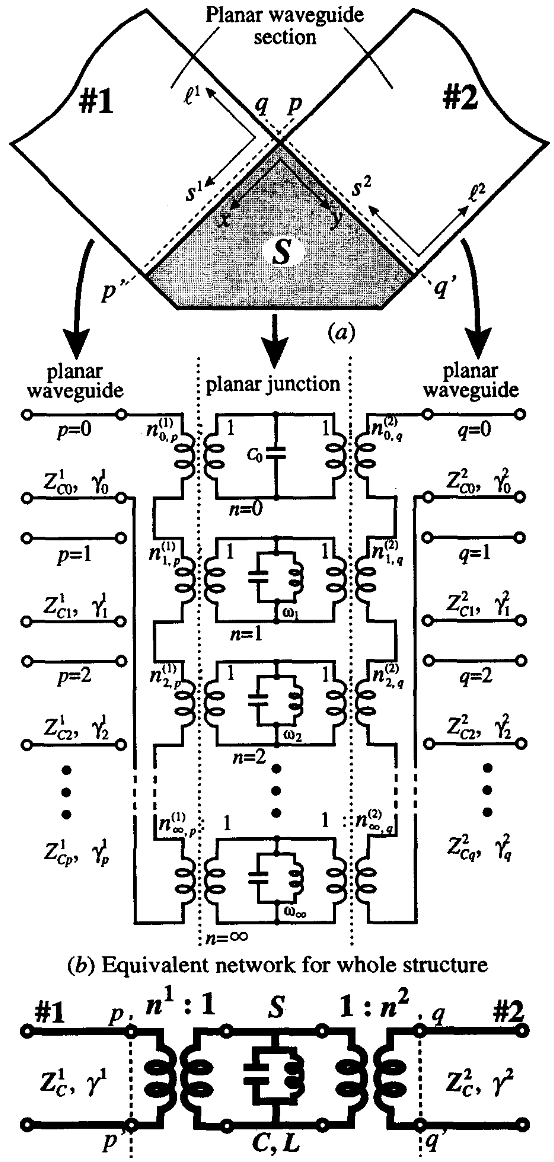


(a)

(b) Equivalent network for whole structure

(c) Vector representation for equivalent network

Fig.2 Right-angle bend with slant-wise corner cut and its equivalent network representaion

Table.1 Eigenmode system

$$\frac{\partial\phi_n}{\partial x^2} + \frac{\partial\phi_n}{\partial y^2} + k_n^2\phi_n = 0 \quad \text{in } S$$

$$\boldsymbol{n} \cdot grad\phi_n = 0 \quad \text{on } C \quad \text{(entire circumference of } S\text{)}$$

$$k_0 = 0,\ k_1 \le k_2 \le \cdots \quad n = 0,\ 1,\ 2,\ \cdots$$

$$\frac{1}{S}\iint_S \phi_n \cdot \phi_m dxdy = \delta_{nm} \qquad \text{(orthonomal)}$$

where S is the area of the planar circuit

type equivalent network shown in Fig.2(b) is derived by mode expansion method, whose vector notation is also given by bold line in Fig.2(c). Network parameter of capacitance, inductance of n-th mode and ideal transformer ratio between n-th mode in the planar junction and p-th mode in the i-th waveguide are given by eq.(3).

$$C_0 = \varepsilon\frac{S}{d} \quad [\mathrm{F}] \qquad L_n = \frac{\varepsilon\mu}{k_n^2}\frac{1}{C_0} \quad [\mathrm{H}]$$

$$n_{np}^{\ i} = \frac{1}{W_i}\int_0^{W_i} \phi_n(x,y) f_p^i(s^i)\, ds^i \tag{3}$$

Therefore, the whole equivalent network for 2-port discontinuity or right-angle bend is given by Fig.2(c) in vector notation.

3. Calculation of eigenmode for square-shaped planar circuit with slant-wise corner cut

Eigenmode for square-shaped planar circuit with slant-wise corner cut as shown in Fig.3(a) are calculated by dividing slant-wisely cutted region(trapezoid) uniformly into M striplines of equal step-length but different width as shown in Fig.3(b). Then each stripline can be represented by equivalent multi-transmission line network as shown in Fig.3(c) and coupling between the width modes including evanescent modes in the striplines #s-1 and #s can be represented by ideal transformer matrix $\boldsymbol{n}^{s\text{-}1,s}$ whose element is given by eq.(4)

$$n_{nm}^{s-1,s} = \frac{1}{W_s}\int_0^{W_s} f_n^{s-1}(x) f_m^s(x)\, dx \tag{4}$$

where $f_n^{s-1}(x)$, $f_m^s(x)$ are width mode function defined by eq.(1). Hence, whole equivalent network or vector notation in bold line is given in Fig.3(c). Therefore, eigenmode for Table.1 can be given by finding the solution which satisfy open boundary condition at both ends. In our analysis, when open boundary at extreme left side (port 1 of waveguide #0) is assumed, input mode admittance matrix at extreme right side (port 2 of waveguide #M) can be systematically calculated as $\tilde{Y}_{in2}^{\ M}$ by cascade connection of stripline and ideal transformer in Fig.3(c) (The detail is explained in Table.2). Because of open boundary at right side, mode current at this port must be zero, which leads to a eigenvalue equation (5).

$$\tilde{i}_2^{\ M} = \tilde{Y}_{in2}^{\ M} v_2^M = 0 \tag{5}$$

In our case, after investigating the convergence behavior of eigenvalue with step-length of striplines constituting trapezoid and number of width mode in the widest waveguide #0, it is determined to be a/100 for the former step-length and take 20 modes for the latter. The eigenvalue up to ka=14 are calculated and shown in Fig.4 as a function of cut parameter C(=c/a). Also using the eigenvector of equation (5), mode voltages and currents at each port of equivalent transmission lines in Fig.3(b) can be calculated step by step based on the equivalent network shown in Fig.3(c), which, combining with the corresponding width mode function, gives 2D field distribution of the corresponding mode. The 3rd mode for various cut parameter are shown in Fig.5.

4. Calculation of the frequency characteristics - determination of the optimum cut -

Based on the above mode calculation, network parameters in Fig.2 and 3 are calculated from eqs.(1),(2). Then, the frequency characteristics of the corner cut right-angle bend for W=5.0[mm], ε_r= 2.62 are calculated, based on the conventional network theory. The

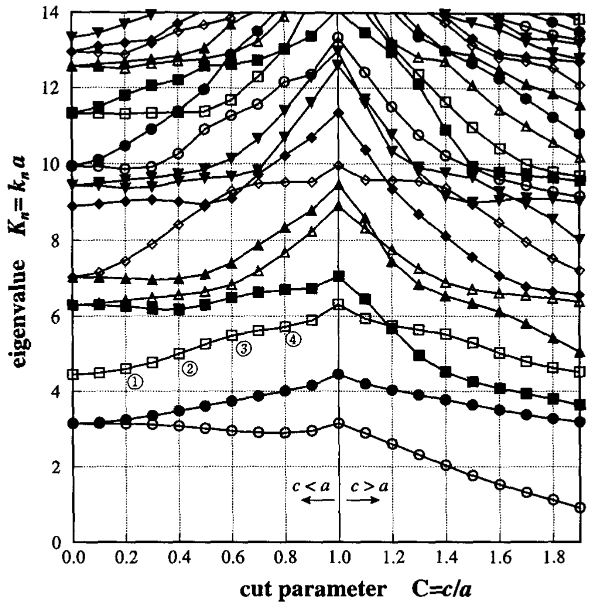
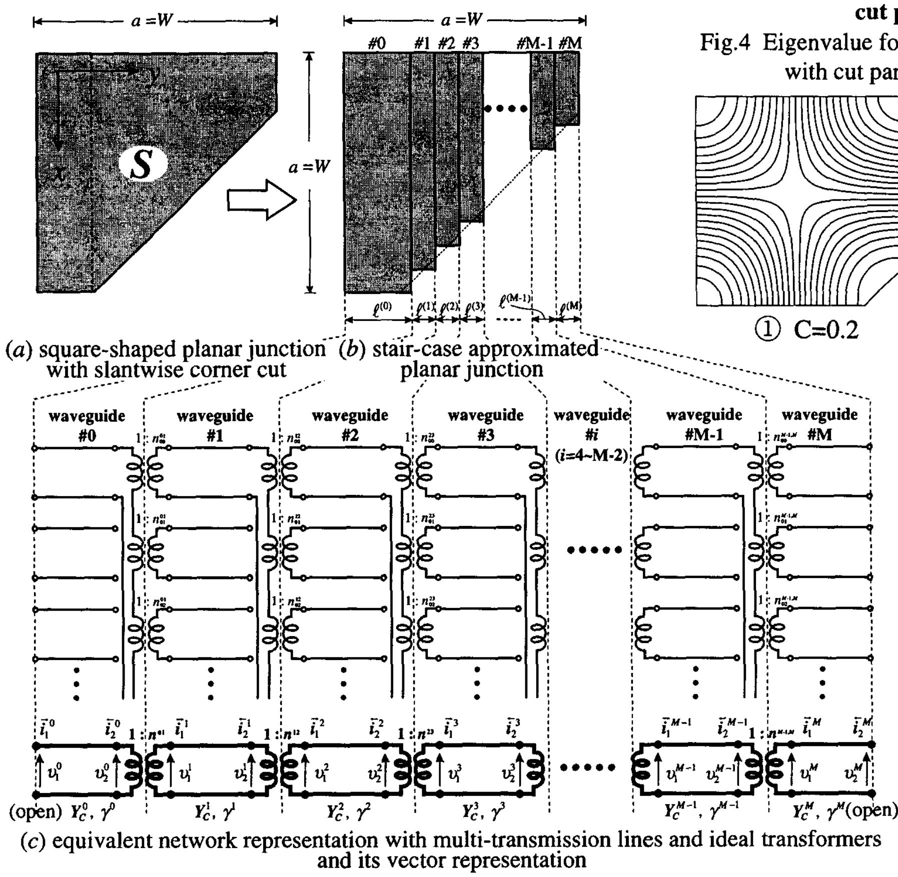


Fig.4 Eigenvalue for slant-wise corner cut with cut parameter C=c/a



(a) square-shaped planar junction with slantwise corner cut (b) stair-case approximated planar junction

(c) equivalent network representation with multi-transmission lines and ideal transformers and its vector representation

Fig.3 Calculation of eigenmode for square-shaped planar junction with slant-wise corner cut by stair-case approximation and the equivalent network representation

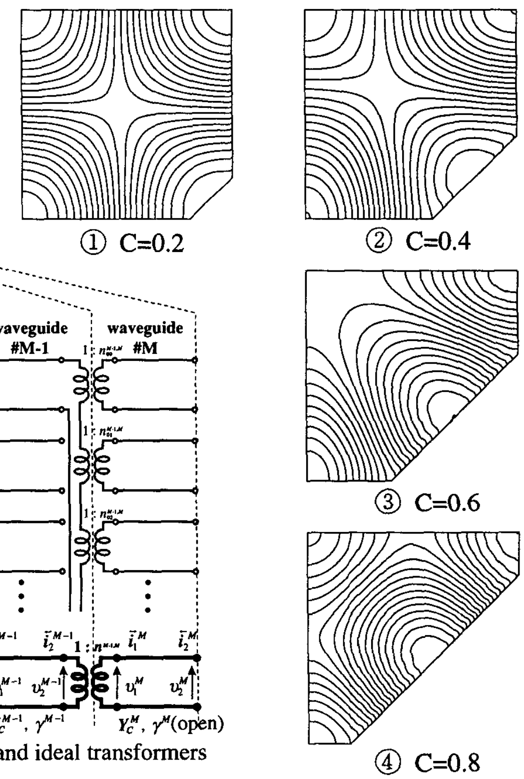


Fig.5 Example of 3rd eigenmode field distribution (stair-case step a/100, p=20)

essential problem in these calculations is how many modes in the waveguide and junction must be taken into consideration (truncation error). After investigating the convergence behavior with number of mode in both regions, it is determined to take up-to 5th waveguide mode and up-to *ka*=20 planar junction mode (26 modes for 0.86 cut parameter). Final calculated results of the frequency characteristics are shown in Fig.6. From this figure it turns out that the optimum cut parameter is about 0.86. The calculation in more detail about 0.86 is given also in Fig.6, which shows that around the cut parameter of 0.86 the larger the cut parameter, the narrower the bandwidth but better transmission.

5. Discussion and conclusion

By general and systematic calculation method, wideband frequency characteristics of right-angle bend with slant-wise corner cut are calculated and compared with that of square-shaped corner cut shown in Fig.8[2]. The realizable limit of transmission vs bandwidth are shown in Fig.7, which demonstrates the former realizes the better performance.

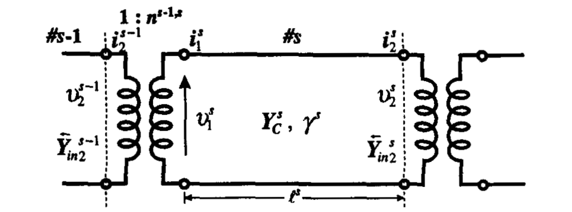
Reference

[1] Reinmut K. Hoffmann, "Handbook of microwave integrated circuit" pp267-309 Artech House (1987)

[2] T. Hiraoka, Y. Tabei, K. Kojima, Hsu, Jui-Pang "Analysis of Stripline Right-angle Bend with Square-shaped Corner cut based on Eigenmode Expansion Method and Foster-type Equivalent Network" APMC'98 WE1B-3

[3] Hsu, Jui-Pang "Analysis method for electromagnetic wave problems edited by E.Yamashita" chap.6, pp226-251 Artech house

Table.2 Calculation method of input mode admittance matrix $\tilde{Y}_{in2}^{M}$ based on iterated calculation of unit structure



$v_1^s = \left(n^{s-1,s}\right)^t v_2^{s-1}$ $\quad Y_T^s = \begin{pmatrix} Y_1^s & Y_2^s \\ Y_2^s & Y_1^s \end{pmatrix}$ $\quad Y_1^s = Y_C \coth \gamma^s \ell^s$

$\tilde{i}_2^{s-1} = n^{s-1,s}\, \tilde{i}_1^s$ $\quad Y_1^s = -Y_C \operatorname{csch} \gamma^s \ell^s$

(multi-port ideal transformer) (multi-transmission line)

$$\tilde{Y}_{in2}^{(s-1)} \Rightarrow \tilde{Z}_{in2}^{(s-1)}$$
$$\Downarrow$$
$$\tilde{Z}_{in1}^{(s)} = \left(n^{s-1,s}\right)^t \tilde{Z}_{in2}^{(s-1)} n^{s-1,s} \Rightarrow \tilde{Y}_{in1}^{(s)} = \left(\tilde{Z}_{in1}^{(s)}\right)^{-1}$$
$$\Downarrow$$
$$\tilde{Y}_{in2}^{(s)} = Y_1^s - Y_2^s\left(\tilde{Y}_{in1}^{(s)} + Y_1^s\right)^{-1} Y_2^s$$

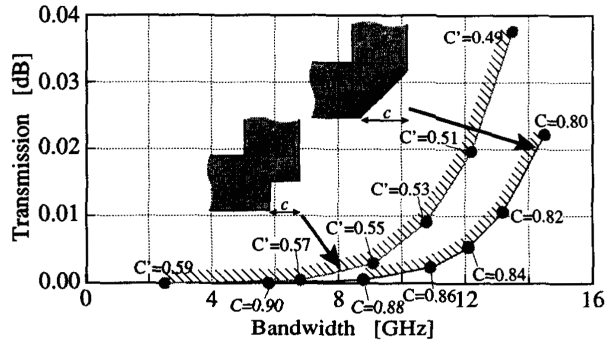


Fig.7 Comparison between slant-wise and square-shaped corner cut - realizable transmission VS bandwidth -

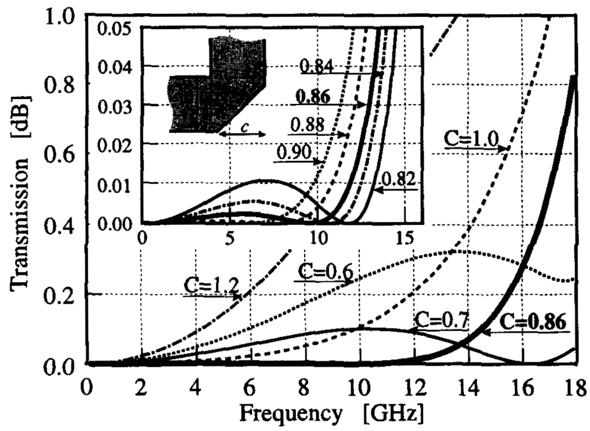


Fig.6 Transmission for slant-wise corner cut with cut parameter C=*c*/*a* and detail (*W*=5.0 [mm], ε_s=2.62)

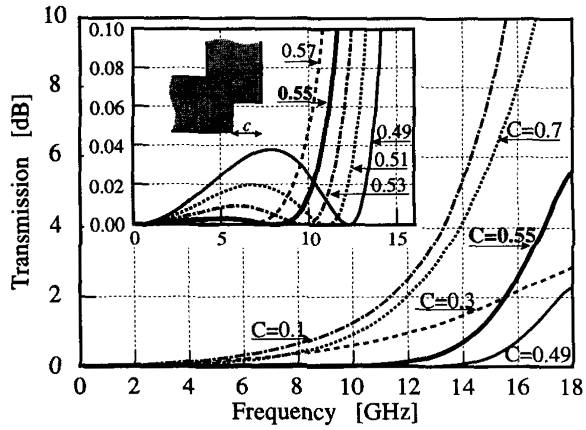


Fig.8 Transmission for square-shaped corner cut with cut parameter C=*c*/*a* and detail (*W*=5.0 [mm], ε_s=2.62)