Equivalent network for rectangular-waveguide H-plane step discontinuity
- Multi-transmission line and Multi-port ideal transformer -

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1. Introduction

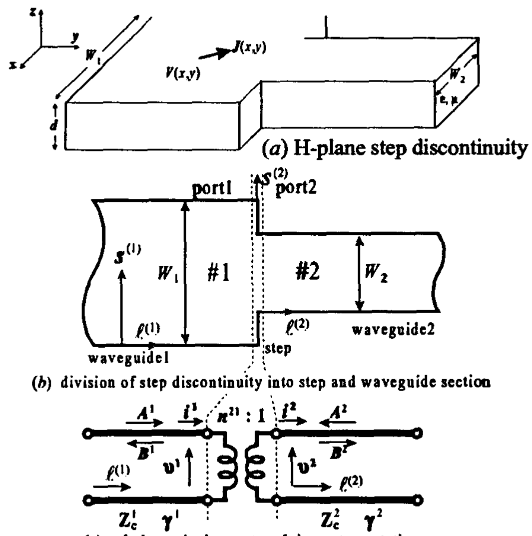
Rectangular-waveguide H-plane step discontinuity as shown in Fig.1(a) is one of the key structures for constructing various H-plane waveguide circuit. In the past, variation method[1] was widely used for analysis because of less computation but reasonable result. Now, abundant computational resources are available, which make mode matching method more practical and effective. Therefore, generalized S-matrix method[2] is useful for calculation of frequency response, but cannot give a good physical picture of field behavior at step. Hence, here modelling the structure by planar circuit and applying modal analysis, the whole equivalent network shown in Fig.1(c) is derived, which consists of multi-transmission line and multi-port ideal transformer with vector notation for waveguide and step section, respectively. The frequency response and field behavior at operation can be easily calculated by this equivalent network. The essential error in this analysis is caused by truncation of mode. Convergence behavior with mode is investigated for practical case, which suggests reasonable mode number. Finally, equivalent network for various H-plane rectangular waveguide circuit are derived, using vector notation of equivalent network.

2. Description of field by planar circuit equations

The step discontinuity shown in Fig.1(a) can be understood as a planar circuit. Assuming TE_{10} excitation, the field component in the circuit are only E_z and $H_t=(H_x,H_y)$, whose relations are given by planar circuit equations (1). where planar circuit voltage and current density are defined by eqs(2).

$$\begin{cases} gradV = -j\omega\mu d J \\ divJ = -j\frac{\omega\varepsilon}{d}V \end{cases} \quad (1) \qquad \begin{aligned} V &\equiv -E_z d \\ J &\equiv H_t \times k \end{aligned} \quad (2)$$

H-plane step discontinuity can be divided into waveguide



(a) H-plane step discontinuity

(b) division of step discontinuity into step and waveguide section

(c) whole equivalent network in vector notation

Fig.1 Step discontinuity and whole equivalent network

(A) General relation as for s,ℓ (K_1, K_2 : arbitrary constant)	
s-dependent equations	ℓ-dependent equations
$\begin{cases} \frac{dV'(s)}{ds} = K_2 J_s'(s) \\ \frac{dJ_s'(s)}{ds} = -\frac{k^2-\beta^2}{K_2}V'(s) \end{cases}$ (A-1)	$\begin{cases} \frac{dV(\ell)}{d\ell} = -\frac{j\omega\mu d}{K_1}J_\ell(s) \\ \frac{dJ_\ell(\ell)}{d\ell} = \frac{\beta^2 K_1}{j\omega\mu d}V(\ell) \end{cases}$ (A-3)
where $J_\ell'(s) = \frac{V'(s)}{K_1}$ (A-2)	where $J_s(\ell) = \frac{-K_2}{j\omega\mu d}V(\ell)$ (A-4)
(B) Present relation as for s,ℓ (K_1=K_2=1 : assumed)	
$\begin{cases} \frac{dV'(s)}{ds} = J_s'(s) \\ \frac{dJ_s'(s)}{ds} = -(k^2-\beta^2)V'(s) \end{cases}$ (B-1)	$\begin{cases} \frac{dV(\ell)}{d\ell} = -j\omega\mu d\, J_\ell(s) \\ \frac{dJ_\ell(\ell)}{d\ell} = -j\frac{\beta^2}{\omega\mu d}V(\ell) \end{cases}$ (B-3)
where $J_\ell'(s) = V'(s)$ (B-2)	where $J_s(\ell) = -\frac{1}{j\omega\mu d}V(\ell)$ (B-4)

Table1 s,ℓ related voltage and current relation

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and step section as shown in Fig.1(b), whose equivalent network is derived in the following.

3. Equivalent multi-transmission line for waveguide

ℓ-s coordinate system is introduced instead of x-y as shown in Fig.2(a). Related voltage and current density can be expressed by eq.(3) using separation of variable form.

$$\begin{cases} V(\ell,s) = V(\ell)\cdot V'(s) \\ J_\ell(\ell,s) = J_\ell(\ell)\cdot J'_\ell(s),\ J_s(\ell,s) = J_s(\ell)\cdot J'_s(s) \end{cases} \quad (3)$$

Substituting eqs.(3) into eqs.(1) and using separation of variable technique, we can derive s-dependent function $V'(s)$, $J'_\ell(s)$, $J'_s(s)$ and ℓ-dependent function $V(\ell)$, $J_\ell(\ell)$, $J_s(\ell)$, which are summarized by box (A) in Table 1, where K_1 and K_2 are arbitrary constants. How to choose K_1-value is related to the definition of mode characteristics impedance. In our analysis, we choose K_1 and K_2 to be unity, then the related equations are given by box (B) in Table1.

Solving eqs.(B-1) under short condition at side wall (s=0,W), p-th width mode function is given by eq(4), which satisfies eq(5) of orthonormal condition.

$$V'_p(s) = \sqrt{2}\sin\frac{p\pi s}{W} \equiv S_p(s) \quad (4) \qquad \frac{1}{W}\int_0^W V'_p(s)V'_q(s)ds = \delta_{pq} \quad (5)$$

Also, p-th mode phase constant is given by eq.(6).

$$\beta_p = \sqrt{k^2 - (p\pi/W)^2} \qquad (p = 1, 2, \cdots) \qquad (6)$$

When p-th mode current $I_p(\ell)$ is defined so that product of mode current and mode voltage gives the corresponding mode power, the mode current is given by $I_p(\ell)$=$J_p(\ell)W$. Then, the transmission line equations along ℓ for p-th mode are derived from eqs.(B-3) in Table1 and given by eqs.(7).

$$\begin{cases} \dfrac{dV_p(\ell)}{d\ell} = -j\dfrac{\omega\mu d}{W}I_p(\ell) & \left(Z_p = j\dfrac{\omega\mu d}{W}\ [\Omega/\mathrm{m}]\right) \\ \dfrac{dI_p(\ell)}{d\ell} = -j\dfrac{\beta_p^2 W}{\omega\mu d}V_p(\ell) & \left(Y_p = j\dfrac{\beta_p^2 W}{\omega\mu d}\ [\mathrm{S/m}]\right) \end{cases} \quad (7)$$

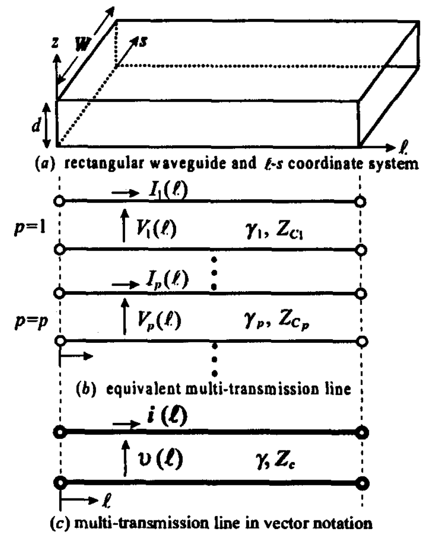
Hence, mode characteristic impedance and mode propagation constant are given by eq.(8)

$$Z_{Cp} = \sqrt{\frac{Z_p}{Y_p}} = \frac{\omega\mu d}{\beta_p W}\ [\Omega] \quad \gamma_p = \sqrt{Z_p Y_p} = j\beta_p\ [\mathrm{rad\ m}] \quad (8)$$

Whole equivalent network for waveguide is shown in Fig.2(b) or by vector notation in Fig.2(c). Column mode voltage and current, characteristic impedance and propagation constant matrix are defined by eqs(9).

$$\begin{cases} \mathbf{Z_C} = diag(Z_{C1}, Z_{C2}, \cdots) & \boldsymbol{\gamma} = diag(\gamma_1, \gamma_2, \cdots) \\ \boldsymbol{\upsilon} = (V_1(\ell), V_2(\ell), \cdots)^t & \boldsymbol{i} = (I_1(\ell), I_2(\ell), \cdots)^t \end{cases} \quad (9)$$

The mode voltage and current distribution along ℓ, $V_p(\ell)$



(a) rectangular waveguide and ℓ-s coordinate system

(b) equivalent multi-transmission line

(c) multi-transmission line in vector notation

Fig.2 Equivalent network for waveguide

and $I_p(\ell)$ are once calculated, voltage and current density distribution in the waveguide are given by eqs.(10).

$$\begin{cases} V(\ell,s) = \sum_{p=1}^{\infty} V_p(\ell,s) = \sum_{p=1}^{\infty} V_p(\ell)S_p(s) \\ J_\ell(\ell,s) = \sum_{p=1}^{\infty} J_{\ell p}(\ell,s) = \sum_{p=1}^{\infty} \dfrac{I_p(\ell)}{W}S_p(s) \\ J_s(\ell,s) = \sum_{p=1}^{\infty} J_{sp}(\ell,s) = \sum_{p=1}^{\infty} -\dfrac{1}{j\omega\mu d}\left(\dfrac{p\pi}{W}\right)V_p(\ell)C_p(s) \end{cases} \quad (10)$$

$$\text{where } S_p(s) = \sqrt{2}\sin\frac{p\pi s}{W},\ C_p(s) = \sqrt{2}\cos\frac{p\pi s}{W} \quad (11)$$

4. Equivalent multi-port ideal transformer for step

Voltage and current density distribution alond s at port1 and 2 of the step shown in Fig.3(a) can be given in eqs.(12) and (13) by the summation of the related mode voltage and current at each port. V_p^1, I_p^1, V_q^2, I_q^2

$$\begin{cases} V^1(s_1) = \sum_{p=1}^{\infty} V_p^1\, S_p^1(s_1) \\ J_\ell^1(s_1) = \sum_{p=1}^{\infty} \dfrac{I_p^1}{W_1}\, S_p^1(s_1) \end{cases} (12) \qquad \begin{cases} V^2(s_2) = \sum_{q=1}^{\infty} V_q^2\, S_q^2(s_2) \\ J_\ell^2(s_2) = \sum_{q=1}^{\infty} \dfrac{I_q^2}{W_2}\, S_q^2(s_2) \end{cases} (13)$$

Also, voltage and current density at the step must satisfy the following continuity condition.

$$J_\ell^1(s_1) = \begin{cases} J_\ell^2(s_2) & (0 < s_2 < W_2) \\ ? & (\text{outside above but in } 0 < s_1 < W_1) \end{cases} \quad (14)$$

$$V^1(s_1) = \begin{cases} V^2(s_2) & (0 < s_2 < W_2) \\ 0 & (\text{outside above but in } 0 < s_1 < W_1) \end{cases} \quad (15)$$

Substituting eqs.(12), (13) into above eqs. and applying orthonormality, following relations are obtained.

$$I_q^2 = \sum_{p=1}^{\infty} n_{qp}^{21} \cdot I_p^1 \ (q = 1,2,\cdots) \quad V_p^1 = \sum_{q=1}^{\infty} n_{qp}^{21} \cdot V_q^2 \ (p = 1,2,\cdots) \tag{16}$$

$$\text{where } n_{qp}^{21} = \frac{1}{W_1}\int_0^{W_1} S_q^2(s_2)\, S_p^1(s_1)\, ds_2 \tag{17}$$

These relation can be expressed by multi-port ideal transformer as shown in Fig.3(*b*), which gives equivalent network for step. When mode voltage and mode current column matrix is defined as v^1, i^1, v^2, i^2 at each port, the relation (16) can be expressed by eqs.(18)[3,4].

$$i^2 = n^{21} \cdot i^1 \qquad v^1 = (n^{21})^t \cdot v^2 \tag{18}$$

where $n^{21} \equiv (n_{qp}^{21})$ transformer ratio matrix

Therefore, multi-port ideal transformer can be expressed by vector notation in Fig.3(*c*).

5. Reduction of field problem to circuit problem

Whole equivalent network for step discontinuity can be expressed by Fig.1(*c*) in vector notation. Thus, field problem is reduced to circuit problem, which can be solved easily by circuit theory. Incident mode voltage and reflected mode voltage column matrix is defined by A^1, A^2, B^1, B^2 at each port, then mode voltage scattering matrix is given by eqs.(19) and (20)[2], where $\bar{\bar{Z}}_{in} = \bar{Z}_{in} Y_C^1$, $\bar{\bar{Y}}_{in} = Z_C^2 \bar{Y}_{in}$ are normalized input mode immittance matrix at port1 and port2.

$$\begin{bmatrix} B^1 \\ B^2 \end{bmatrix} = \begin{bmatrix} S_V^{11} & S_V^{12} \\ S_V^{21} & S_V^{22} \end{bmatrix} \begin{bmatrix} A^1 \\ A^2 \end{bmatrix} \tag{19}$$

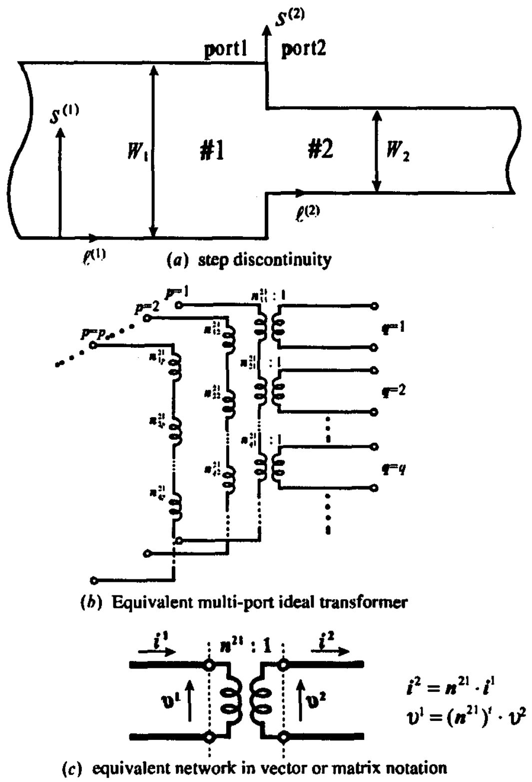
$$\begin{aligned} S_V^{11} &= \left(\bar{\bar{Z}}_{in} + I\right)^{-1}\left(\bar{\bar{Z}}_{in} - I\right) \quad & S_V^{22} &= \left(I + \bar{\bar{Y}}_{in}\right)^{-1}\left(I - \bar{\bar{Y}}_{in}\right) \\ S_V^{21} &= Z_C^2\, n\, Y_C^1\left(I - S_V^{11}\right) \quad & S_V^{12} &= n^t\left(I + S_V^{22}\right) \end{aligned} \tag{20}$$

6. Numerical Example

Calculated results of symmetric step discontinuity shown in Fig.1(*a*) (W_1=20mm, W_2=10mm, d=5mm with air) are shown in Fig.4, where p and q mean number of mode considered at W_1 and W_2 with $p/q = W_1/W_2$. Fig.4(*a*) and (*b*) show that frequency response will converge at $p \approx 40$; but Fig.4(*d*) shows that p=40 is not enough but p=100 seems to be enough for realizing continuity of the voltage and current density distribution at the step. Fig.4(*c*) shows voltage distribution of zero phase at 20GHz with p=100. Far from the step, necessary number of mode for describing the field is reduced due to evanescent character of higher mode.

7. Conclusion

1. Equivalent network for H-plane step is derived from modal analysis and expressed by multi-port ideal transformer or vector notation as shown in Fig.3(*b*) or (*c*).



(*a*) step discontinuity

(*b*) Equivalent multi-port ideal transformer

(*c*) equivalent network in vector or matrix notation

Fig.3 Step and its equivalent multi-port ideal transformer

2. Vector notation of equivalent network for waveguide was proposed as in Fig.2(*c*). Then, whole equivalent network for step discontinuity is given by Fig.1(*c*).

These conclusion are consistent with conventional results[2,3,4] and help to understand mode behavior at step. Finally, equivalent network for various H-plane waveguide circuit are derived in Fig.5 as example.

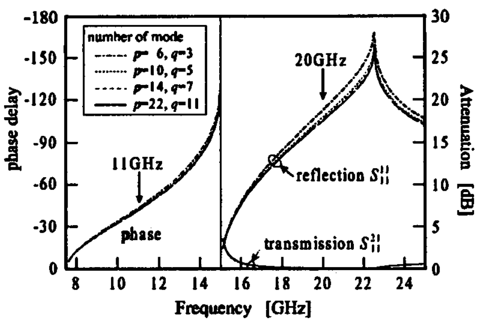
Reference

(1) N. Marcuvitz, ed. "Waveguide Handbook" pp168-172 Mcgraw-hill first edition 1951

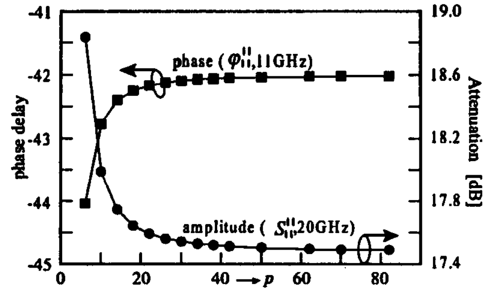
(2) Yi-Chi Shih "The mode matching method" Chap.9 of Numerical technique for microwave and millimeter-wave pas sive structure ed. by T.Itoh, John Wiley & Sons. Inc. 1989

(3) Ferdinando Alessandri, Giancarlo Bartolucci, Roberto Sorrentino"Admittance Matrix Formulation of Waveguide Dis continuity Problems: Computer-Aided Design of Branch Guide Directional Couplers" IEEE vol.MTT-36, No.2, pp394-403

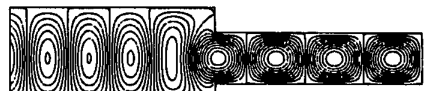
(4) A.Weisshaar, M.Mongiardo, V.K.Tripathi "CAD oriented equivalent circuit modeling of step discontinuities in rectangular waveguides", IEEE Microwave and guided letters, vol.6 No.4, pp171-173 1996



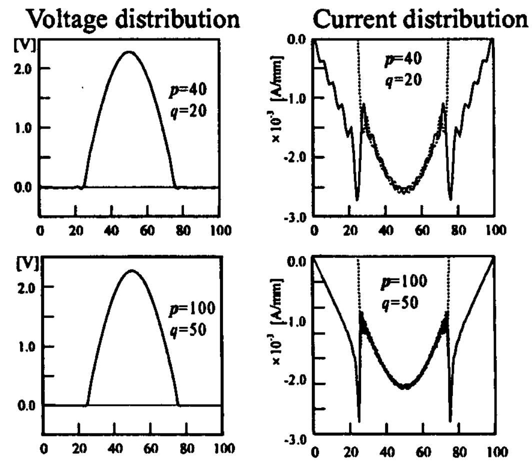
(*a*) Convergence of frequency characteristics



(*b*) Convergence of phase and amplitude

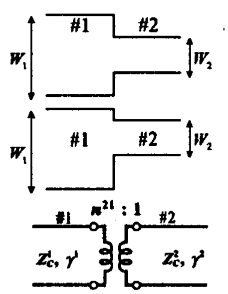


(*c*) Voltage distribution at operation of 0 phase

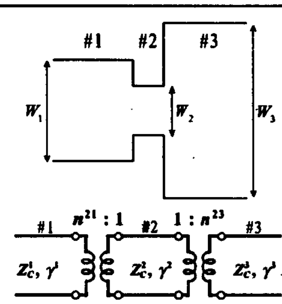


(*d*) Continuity of the voltage and current at step (solid line at port1 and dotted line at port2)

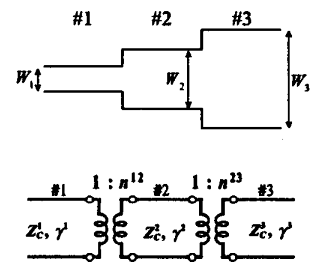
Fig.4 Calculated results and convergence behavior with mode for step discontinuity (W_1=20mm, W_2=10mm, d=5mm, center connection, specified frequency : 20GHz)



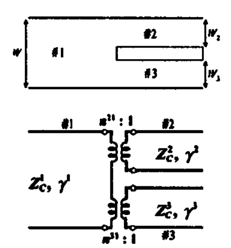
(*a*) Symmetric/Asymmetric step



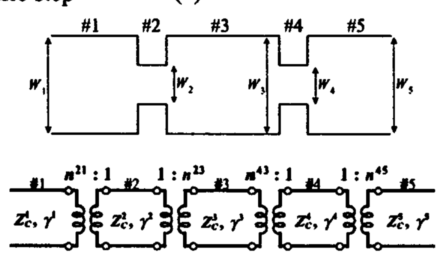
(*b*) Inductive window



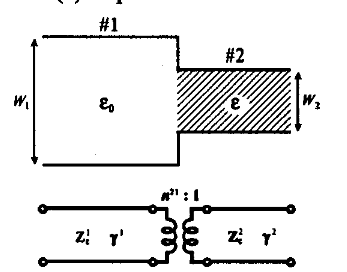
(*c*) Impedance transformer



(*d*) power divider



(*e*) waveguide coupled cavity



(*f*) step discontinuity with different dielectric

Fig.5 Various H-plane rectangular waveguide circuit with equivalent network representation