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Measurement of Electromagnetic Field Distribution in Waveguide Based on Analogy between H-Plane Waveguide- and Trough-Type Planar Circuit

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SUMMARY A detailed investigation of the electromagnetic field distributions inside waveguide circuits is useful for physical understanding, studies of electromagnetic coupling effects for EMC and EMI and for optimization of waveguide circuit designs. In this paper, we describe how to calculate and measure the two-dimensional electromagnetic field distributions inside waveguide-type planar circuits, making use of an analogy between H-plane waveguide- and trough-type surface-wave planar circuits. The measurement results are in good agreement with the results of the numerical analysis based on the normal mode expansion method.

key words: *waveguide circuit, electromagnetic field distribution, probe field mapping*

1. Introduction

The rapid development of various electromagnetic circuits in the microwave and millimeter-wave ranges has made the rigorous analysis of these circuits increasingly important.

It is considered that a detailed investigation of two-dimensional electromagnetic (EM) field distribution, in addition to the input/output frequency characteristics of microwave circuits, is useful not only for the optimization and adjustment of the electromagnetic circuits, but also for predictions in the theoretical analysis of more complicated discontinuity problems. As for the method of numerical analysis, much research has already been reported [1]-[3]. However, as far as the authors know, no attempt has been made to analyze frequency characteristics of the EM field distributions inside H-plane waveguide-type planar circuits. Recently, for measurement of two-dimensional field distributions in MMIC fabricated on GaAs and $LiTaO_3$ substrates, new contactless test techniques using electro-optic probe based on the Pockels effect have been reported. However, such techniques are applicable only for special substrates [4],[5]. Since H-plane waveguide type planar circuits are a perfectly closed systems, direct measurement of the EM field distributions is very difficult. To obtain the EM field distributions inside H-plane waveguide circuits, a special technique is required. In this paper, we will explain how to calculate and measure the two-dimensional EM field distributions inside the planar circuits, making use of an analogy existing between H-plane waveguide- and trough-type surface-wave planar circuits. The technique itself is based on a two-dimensional electric probe measurement system controlled by microcomputer [6].

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2. Measurement Method Based on Analogy between H-Plane Waveguide- and Trough-Type Planar Circuit

2.1 Structure and Basic Equations

Before starting the derivation of the basic relations, let us briefly review the planar circuit structure treated here. We consider a structure of (a) a H-plane waveguide-type planar circuit and (b) a trough-type surface-wave planar circuit having the same metal-wall boundary conditions as shown in Figs. 1(a) and 1(b). The waveguide-type planar circuit is uniform in the x-y plane and has a closed system. The trough-type planar circuit is uniform in the x-y plane and has an upper

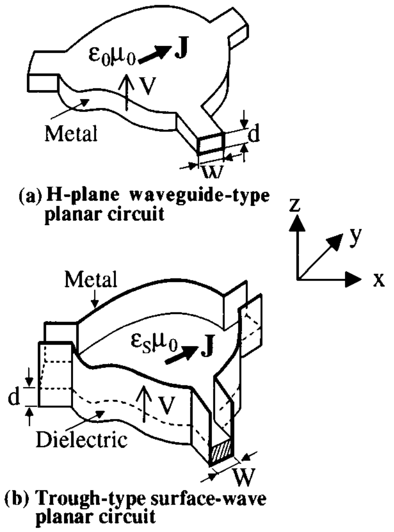
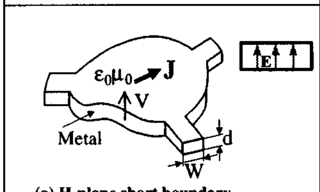
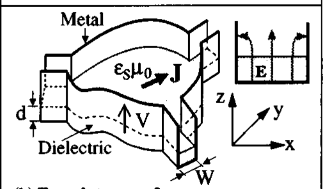


Fig. 1 Strucutre of planar circuit.

Table 1 Planar circuit equations and Helmhotz equation.

Short boundary Planar Circuit	Trough-type Surface Planar Circuit
 (a) H-plane short boundary planar circuit	 (b) Trough-type surface-wave planar circuit
Electromagnetic Field Components	
$\mathbf{E}=(0, 0, E_z)$ $\mathbf{H}=(H_x, H_y, 0)$	$\mathbf{E}=(E_x, E_y, E_z)$ $\mathbf{H}=(H_x, H_y, 0)$
Planar Circuit Voltage and Current	
$V(x,y)=-E_z(x,y)\cdot d$ [V] $\mathbf{J}(x,y)=\mathbf{H}_t(x,y)\times\mathbf{k}$ [A/m]	$V^E(x,y)=-E_z(x,y)\cdot d$ [V] $\mathbf{J}^E(x,y)=\mathbf{H}_t(x,y)\times\mathbf{k}$ [A/m]
Planar Circuit Equations	
$\mathrm{grad}\,V=-jX_n\mathbf{J}$ $X_n=\frac{k_0^2 d}{\omega\varepsilon_0}$ [Ω] $\mathrm{div}\,\mathbf{J}=-jB_nV$ $B_n=\frac{\omega\varepsilon_0}{d}$ [S/m²]	$\mathrm{grad}\,V^E=-jX_n^E\mathbf{J}^E$ $X_n^E=\frac{\beta_{tn}^2}{\omega\varepsilon_0}$ [Ω] $\mathrm{div}\,\mathbf{J}^E=-jB_n^EV^E$ $B_n^E=\frac{\omega\varepsilon_0}{d}$ [S/m²]
Two Dimensional Helmholtz Equation(same boundary condition)	
$\nabla_t^2V(x,y)+k_0^2V(x,y)=0$ $V=0$ on C_s $\partial V/\partial n=0$ on C_w $k_0^2=\omega^2\varepsilon_0\mu_0$	$\nabla_t^2V^E(x,y)+\beta_{tn}^2V^E(x,y)=0$ $V^E=0$ on C_s $\partial V^E/\partial n=0$ on C_w $\tan\sqrt{\varepsilon_s k_0^2-\beta_{tn}^2}\cdot d=\frac{\varepsilon_s\sqrt{\beta_{tn}^2-k_0^2}}{\sqrt{\varepsilon_s k_0^2-\beta_{tn}^2}}$

open structure, and its specific dielectric constant ε_s varies as a function of the height coordinate z, and its field decays exponentially in the height direction. The electric fields have E_z and $\boldsymbol{E}_t$ components, and the magnetic fields have only $\boldsymbol{H}_t$ component. That is, the field component in both planar circuits are

$$\boldsymbol{E}=(\boldsymbol{E}_t, E_z) \quad \boldsymbol{H}=(\boldsymbol{H}_t, 0).$$

To relate the electromagnetic field analysis with the circuit theory, we define the mode voltage and mode current. Voltage and currents densities in the planar circuit are defined as

$$V(x, y)=-E_z d, \quad \boldsymbol{J}(x, y)=(Hy, Hx). \tag{1}$$

The basic planar circuit equations for both planar circuits and the Helmholtz equation with regard to voltage are derived from Maxwell's equations, respectively [3],[7]. Based on the scalar voltage and two-dimensional vector current defined properly by (1), the planar circuit equations are presented in Table 1. All the expressions for the field distributions and related equations are also summarized in Table 1. The frequency responses for the planar circuits are obtained in principle by solving these planar circuit equations under given boundary and excitation conditions. In these relations, both planar circuits have essentially the same planar circuit equations and boundary conditions except for the lateral wavenumber k_0 and β_t. Therefore, referring to the $\omega-\beta$ diagram shown in

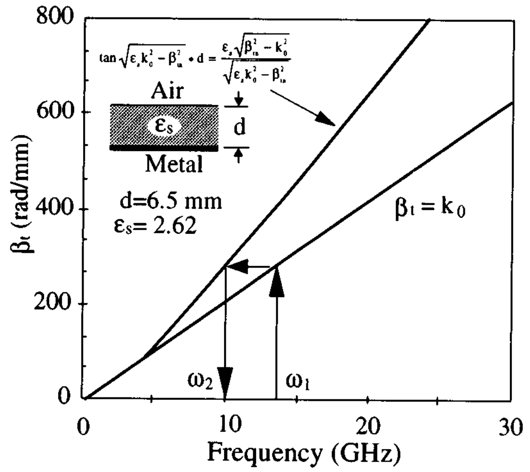


Fig. 2 Dispersion curve of dielectric slab.

Fig. 2, if $k_0(\omega_1)=\beta_t(\omega_2)$ in both planar circuits, the input/output characteristics and the EM field distributions have the same property.

In accordance with the relation in this analogy (so called frequency translation), it is seen that the properties at frequency ω_1 in Fig. 1 (a) correspond to those at frequency ω_2 in (b). Thus, the difficulty of measuring the EM fields in the closed-system structure of (a) can be avoided if we measure, instead, the corresponding fields in the trough-type circuit of (b), because the open-type structure of the latter circuit permits easy measurement.

2.2 Electric Field Distribution inside Planar Circuits

When a planar circuit is excited by TE_{10} mode incidence at the input/output coupling waveguide, the EM field distribution is described using Green's function $\boldsymbol{G}(\boldsymbol{r};\ \boldsymbol{r}_0)$ at (x, y, d) due to a unit current density source located at the source point $\boldsymbol{r}_0$

$$\nabla_t^2 G(\boldsymbol{r};\ \boldsymbol{r}_0)+k^2G(\boldsymbol{r};\ \boldsymbol{r}_0)=-\delta(\boldsymbol{r}-\boldsymbol{r}_0), \tag{2}$$

where $\boldsymbol{G}(\boldsymbol{r};\ \boldsymbol{r}_0)$ satisfies the following boundary conditions:

$\boldsymbol{G}=0$ on C (Periphery)

$\partial\boldsymbol{G}/\partial\boldsymbol{n}=0$ on C_w (coupling port parts).

For a surface current distribution $\boldsymbol{J}_s$ on the coupling port connected in the planar circuit, two-dimensional normal component $V(x, y)=-E_z(x, y)d$ is given by

$$V(x, y)=Z\int G(\boldsymbol{r};\ \boldsymbol{r}_0)\cdot \boldsymbol{J}_s dxdy, \tag{3}$$

where

$$Z=j\frac{k_0^2 d}{\omega\varepsilon_0} \text{ or } j\frac{\beta_t^2 d}{\omega\varepsilon_0},$$

$$J_s^{(i)}(s^{(i)})=I_q^{(i)}/W^{(i)}\cdot f_q^{(i)}(s^{(i)})$$

Table 2 Normal mode function.

$$\frac{\partial^2 \varphi_n}{\partial x^2} + \frac{\partial^2 \varphi_n}{\partial y^2} + k_n^2 \varphi_n = 0 \quad \text{in } S$$

$$\varphi_n = 0 \quad \text{on } C \text{ (periphery)}$$

$$\frac{\partial \varphi_n}{\partial n} = 0 \quad \text{on } C_W \text{ (Coupling port parts)}$$

$$0 < k_1 \leq k_2 \leq \cdots$$

$$\frac{1}{S}\iint_S \varphi_n \cdot \varphi_m \, dx dy = \delta_{nm}$$

S: Area of planar circuit

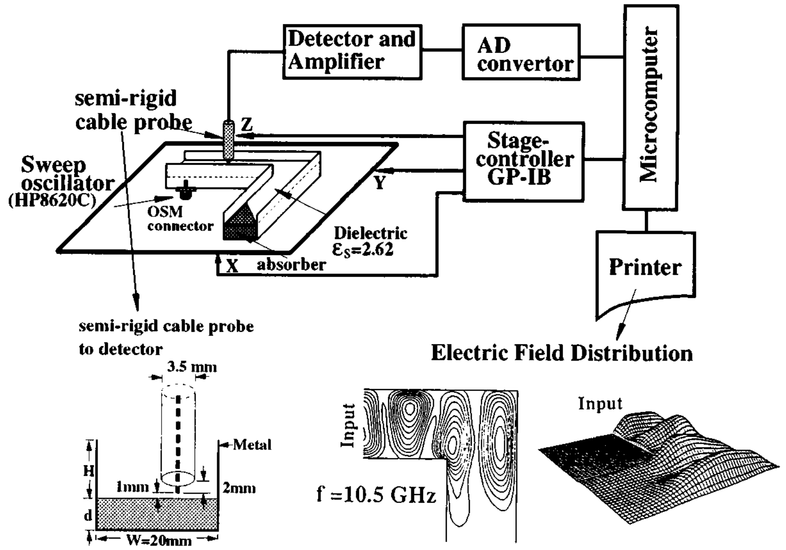


Fig. 3 Block diagram of two-dimensional electric field measurement set up.

for the H-plane waveguide and trough guides, respectively. β_t is the propagation constant for the metal/dielectric/air slab waveguide shown in Fig. 2.

Since $\boldsymbol{G}(\boldsymbol{r}\,;\,\boldsymbol{r}_0)$ can be expanded in terms of the set of eigenfunctions having the same boundary conditions, then, $E_z(x, y)$ can be calculated using

$$E_z(x, y) = \sum_{n=1}^{\infty}\left[-\frac{j\omega\mu}{\omega^2\varepsilon\mu - k_n^2}\cdot\frac{1}{S}\sum_{j=1}^{m}\sum_{q=1}^{\infty} n_{qn}^{(j)} I_q^{(j)}\right] \cdot \varphi_n(x, y), \tag{4}$$

where k_n and $\varphi_n(x, y)$ are the eigenvalue and normal mode function given in Table 2. $n_{n,q}^{(j)}$ is the ideal transformer ratio between the n-th planar mode and q-th mode of the j-th transmission line and given by

$$n_{qn}^{(i)} = \frac{1}{W^{(i)}}\int_0^{W(i)} \varphi_n(x_0, y_0)\cdot f_q^{(i)}(s^{(i)})\, ds^{(i)}, \tag{5}$$

$$f_q^{(i)}(s^{(i)}) = \sqrt{2}\sin\frac{q\pi s^{(i)}}{W^{(i)}} \text{ (Eigenfunction of transmission-line)}.$$

In order to calculate the field distribution accurately using this method, we need infinitely many eigenmodes in the planar circuit as well as infinitely many nonpropagating higher modes in the transmission lines in the above expressions. In practice, however, this is impossible. Therefore, we take into account 50 eigenmodes in the planar circuit and 4 higher-order modes in the waveguide, after investigating the convergence behavior with a number of modes in the planar circuit and waveguide in the practical case [7]-[8].

2. 3 Measurement Setup

Figure 3 shows a block diagram of the setup for measuring the electromagnetic field distributions inside the trough-type planar circuits. As described above, since the field component $E_z(x, y)$ is always vertical to dielectric sheet, we set a probe antenna upright over the dielectric sheet. The measurements are carried out by sliding the electric probe above the x-y plane of the dielectric sheet, keeping it at a constant height. This probe-antenna is mounted on a movable rack mechanism that maintains a constant spacing between the probe tips and dielectric sheet even when the probe moves above the x-y plane. The computer-controlled stepping motor is used and the EM fields are sampled using a 12 bit A/D convertor and stored in memory (Scanning minimum steps of 0.1 mm in the x- and y-directions were employed, and hence, 10000 field values were sampled in each measurement per cycle).

Furthermore, the probe-antenna is formed by protruding the center conductor of the semi-rigid cable into the enclosure. For reduction of the unwanted field disturbance caused by the electric probe, the probe of the detector can only be coupled weakly into the waveguide circuits. The matched load connected at the end of the waveguide circuit is formed by a tapered lossy material absorber so as to realize no reflection. However, due to the imperfection of the matched load, a small amount of frequency dependent reflection occurs and this mismatched termination affects the accuracy of the measurement.

3. Numerical and Experimental Results

In order to confirm the validity and usefulness of the field measurement system, we show calculation and measurement examples of the frequency dependent field distributions for H-plane-and trough-type waveguide circuits. The dimensions of the trough-type waveguide measured here are dielectric line-width W $=20$ mm, dielectric sheet thickness $d=6.5$ mm, height of sidewall $H=20$ mm, and relative dielectric constant $\varepsilon_s=2.62$.

3. 1 Frequency Response of EM Field Distribution of Right Angle Corner-Bend

Referring to the $\omega-\beta$ diagram as shown in Fig. 4(a) and using the relation of the frequency translation, this frequency (about 10.5 GHz) for the waveguide corresponds to the frequency (8.5 GHz) for the trough-type guide. And therefore, the frequency characteristic at the frequency (10.5 GHz) for the waveguide-type(a) corresponds to that at the frequency (8.5 GHz) for the trough-type(b). Figures 4(b) and 4(c) show calcu-

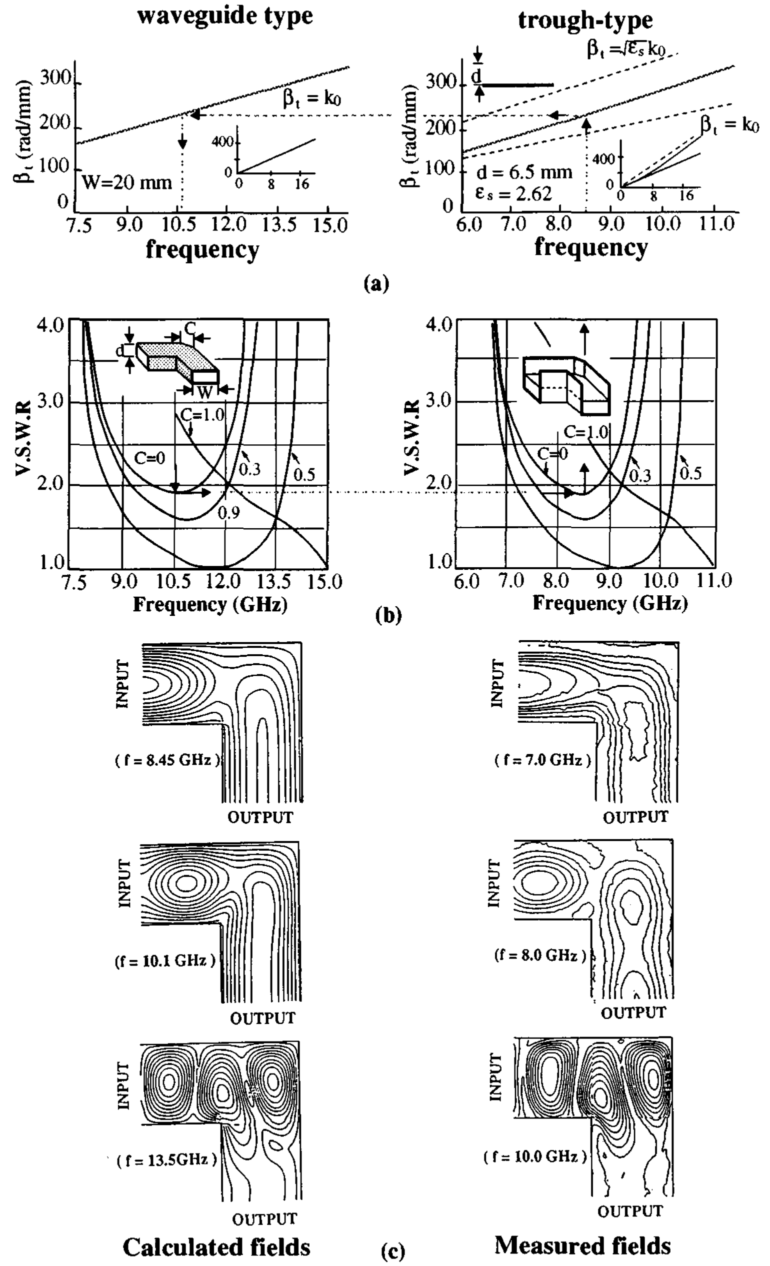


Fig. 4 Frequency responses of electric field distribution for right angle corner bend ((a) Dispersion curve, (b) Frequency characteristics, and (c) Field distributions).

lated frequency characteristics of $|S_{i,j}|^2$ with corner-cut $C(=C/W)$, and electric field contour plots for the H-plane waveguide right angle corner bend. The measurement results of frequency-dependent electric field distributions for the $C=0$ case are compared with the numerical results based on the normal mode expansion method. The analytical and measurement results show the same frequency dependent field distributions.

3.2 Electric Field Distribution of H-Plane T-Junction

Figure 5 also shows the calculated and measured frequency characteristics of $|S_{i,j}|^2$ and the electric field distributions for the H-plane T-junction. When excited from port 1 at the dominant mode, it is found that the power is delivered equally with about 38% between the two output ports 2 and 3 in the frequency range of $f=9$-12 GHz, and the power is almost totally reflected at frequencies below $f=9$ GHz and frequencies above $f=13.5$ GHz. Clearly, good agreement has been obtained. Thus, a physical understanding of frequency characteristics in the T-junction is given by the patterns of electric field distributions. Very good agreement was also found vindicating the analogy between the closed and open structures.

4. Conclusion

A method for calculating and measuring the two-dimensional EM field distributions inside both planar circuits was proposed, making use of the analogy between the H-plane waveguide- and trough type surface-wave planar circuits having the same boundary

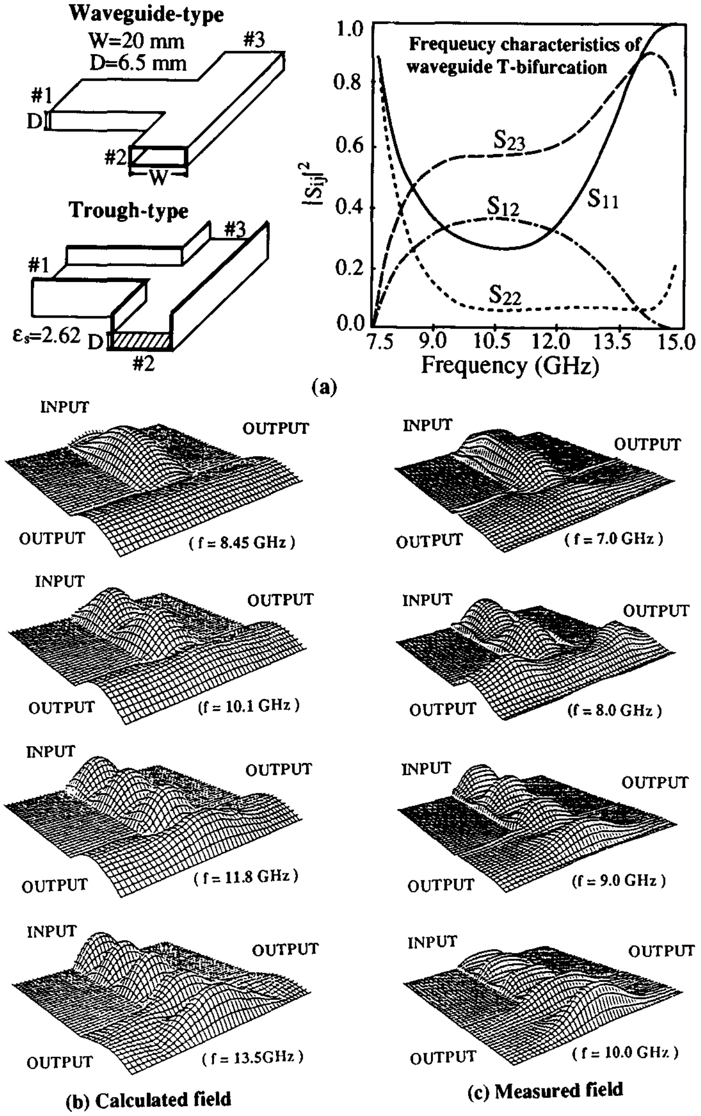


Fig. 5 Frequency responses of electric field distribution for T-junction ((a) Frequency characteristics, (b) Calculated fields, and (c) Measured fields).

conditions. The agreement between theory and measurement was quite satisfactory. Thus, the proposed measurement system offers a valid means for predictions in the theoretical analysis of more complicated discontinuity problems. The method of analysis and measurement described here will be also applied to various H-plane structures. In the future, we intend to measure electromagnetic field distributions in more complex discontinuity problems such as E-plane planar circuits.

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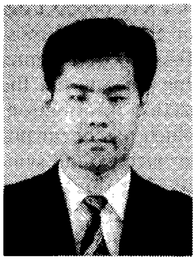

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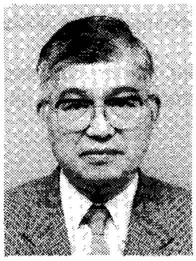

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