Calculation of Propagation Eigenmode for Stripline with Finite Thickness based on Equivalent Network along Horizontal or Vertical Direction

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***Abstract* -- Exact propagation eigenmode for stripline with finite thickness is important for practical design of MIC. So far propagation eigenmode for stripline with zero thickness is given exactly by conformal transformation method, but not for that with finite thickness. In order to treat this problem analytically, lateral equivalent network in general is derived and applied to obtain equivalent network of stripline structure along horizontal or vertical direction. Equivalent network for two direction are different, but gives same propagation eigenmode, which demonstrate the usefulness of the derived lateral equivalent network.**

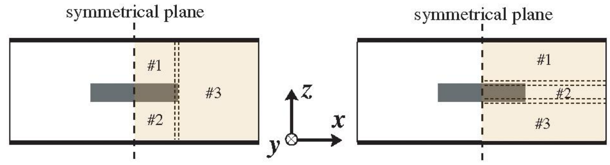
I. INTRODUCTION

Stripline is an important transmission line for MIC system. Hence, exact propagation eigenmode (propagation constant and field distribution) for stripline with finite thickness is strongly needed for rigorous analysis and synthesis of practical stripline circuit. So far propagation eigenmode for stripline with zero thickness is given exactly by conformal transformation method, but not for that with finite thickness. In order to treat this problem analytically, lateral equivalent network is derived by mode analysis/planar circuit theory/mode matching method[1],[2],[3] and practically applied to the calculation of stripline with zero thickness. Agreement of the calculated result with exact one shows the validity of the derived lateral equivalent network[4].

In this paper, lateral equivalent network is applied to obtain equivalent network of stripline structure along horizontal or vertical direction, where stripline cross-section is divided along horizontal direction or along vertical direction as shown in Fig.1 (a) or (b). Each equivalent network for two directions and formulation of the eigenvalue equation is different regardless of the same structure, but must give the same propagation eigenmode. Practically eigenmodes are calculated by two ways. Calculated propagation constant and field distribution are compared and agreed, which demonstrate the usefulness of the derived lateral equivalent network.

II. LATERAL EQUIVALENT NETWORK

In order to calculate propagation eigenmode based on lateral equivalent network, cross-section of stripline is divided into three uniform regions and two step discontinuties as shown in Fig.1 a) or b), which is again shown in Fig.2 and Fig.3. Then each region becomes planar circuit whose field can be given by solution of TE/TM height mode corresponding planar circuit equations. By the way x-y dependent 2D planar circuit solution becomes x dependent 1D transmission line solution because of $e^{-j\beta_{//}y}$ dependency along waveguide. Further more



a) partition along horizontal direction b) partition along vertical direction

Fig.1 Stripline structure and partition of its half cross-section by two ways

each height mode of neighboring planar circuit is coupled, and their coupling can be expressed by ideal transformer bank and mode conversion current source.

A. Planar circuit equations

When TE/TM height mode function $f(z)$, $g(z)$, $h(z)$ and eigenvalue corresponding to transverse propagation contant are given, planar circuit equations for each mode are given below.

$$\begin{cases} gradV(x,y) = -jXJ(x,y) \\ divJ(x,y) = -jBV(x,y) \end{cases} \qquad (1)$$

where planar voltage V(x,y)/current **J**(x,y), planar series reactance X/planar shunt susceptance B and planar characteristic admittance Y_c/propagation constant β_t are properly defined in Table I. Height mode function for the structure of Fig.2 and Fig.3 are given in Table II and III, respectively.

B. Lateral transmission line for Uniform Regions

x-y dependent 2D planar circuit equation given by eq.(1) becomes x dependent 1D transmission line equation because of $e^{-j\beta_{//}y}$ dependency along waveguide.

$$\begin{cases} \dfrac{dV(x)}{dx} = -jXJ_x(x) \\ \dfrac{dJ_x(x)}{dx} = -j\dfrac{(\beta_\perp)^2}{X}V(x) \end{cases} \qquad (2)$$

Hence, equivalent network for three uniform regions in Fig.2(a) and Fig.3(a) are given by multi-transmission line of TE/TM mode as shown in Fig.2(b) and Fig.3(b), where propagation constants and characteristic admittance in parallel(y) and normal(x) direction are given by eq.(3), respectively.

$$Y_{c//} = Y_c\frac{\beta_{//}}{\beta_t} \quad Y_{c\perp} = Y_c\frac{\beta_\perp}{\beta_t} \quad \beta_\perp = \sqrt{\beta_t^2 - \beta_{//}^2} \qquad (3)$$

TABLE I Definition and Parameter for Planar Circuit

TE(H) mode	TM(E) mode
$V_m^H(x,y) = H_{zm}^H(x,y)\cdot d[A]$ $\mathbf{J}_m^H(x,y) = \mathbf{k}\times\mathbf{H}_{tm}^H(x,y)[V/m]$	$V_n^E(x,y) = -E_{zn}^H(x,y)\cdot d[V]$ $\mathbf{J}_n^E(x,y) = \mathbf{H}_{tn}^H(x,y)\times\mathbf{k}[A/m]$
$B_m^H = \omega\mu_0/d[S/m^2]$ $X_m^H = (\beta_{tm}^H)^2\cdot d/\omega\mu_0[S]$	$B_n^E = \omega\varepsilon_0/d[S/m^2]$ $X_n^E = (\beta_{tn}^E)^2\cdot d/\omega\varepsilon_0[\Omega]$
$Y_{cm}^H = \sqrt{\dfrac{B_m^H}{X_m^H}} = \dfrac{\omega\mu_0}{\beta_m^H}\dfrac{1}{d}[\Omega/m]$ $\beta_{tm}^H = \sqrt{X_m^H B_m^H}[rad/m]$	$Y_{cn}^E = \sqrt{\dfrac{B_n^E}{X_n^E}} = \dfrac{\omega\varepsilon_0}{\beta_n^E}\dfrac{1}{d}[S/m]$ $\beta_{tn}^E = \sqrt{X_n^E B_n^E}[rad/m]$

C. *Equivalent network for step discontinuity*

Continuity of tangential field at discontinuity gives following mode coupling equations in matrix form.

(1) *Coupling for horizontal partition case (Fig.2)*

$$
\begin{cases}
V^{1H,2} = (F^{3H1H})^t V^{3H,1} & \bar{J}_\perp^{3H,1} - i^{3H,1} = F^{3H1H}(\bar{J}_\perp^{1H,2} - i^{1H,2}) \\
V^{2H,2} = (F^{3H2H})^t V^{3H,1} & \bar{J}_\perp^{3H,1} - i^{3H,1} = F^{3H2H}(\bar{J}_\perp^{2H,2} - i^{2H,2})
\end{cases}
$$
$$
\begin{cases}
V^{3E,1} = (F^{1E3E})^t V^{1E,2} + (F^{2E3E})^t V^{2E,2} \\
\bar{J}_\perp^{aE,2} - i^{aE,2} = F^{aE3E}(\bar{J}_\perp^{3E,1} - i^{3E,1}) \quad (a=1,2)
\end{cases}
\tag{4}
$$

(2) *Coupling for vertical partition case (Fig.3)*

$$
\begin{cases}
V^{2H,1} = (F^{1H2H})^t V^{1H,2} & \bar{J}_\perp^{1H,2} - i^{1H,2} = F^{1H2H}(\bar{J}_\perp^{2H,1} - i^{2H,1}) \\
V^{2H,2} = (F^{3H2H})^t V^{3H,1} & \bar{J}_\perp^{3H,1} - i^{3H,1} = F^{3H2H}(\bar{J}_\perp^{2H,2} - i^{2H,2})
\end{cases}
$$
$$
\begin{cases}
V^{1E,2} = (F^{2E1E})^t V^{2E,1} & \bar{J}_\perp^{2E,2} - i^{2E,1} = F^{2E1E}(\bar{J}_\perp^{1E,2} - i^{1E,2}) \\
V^{3E,1} = (F^{2E3E})^t V^{2E,2} & \bar{J}_\perp^{2E,2} - i^{2E,2} = F^{2E3E}(\bar{J}_\perp^{3E,1} - i^{3E,1})
\end{cases}
\tag{5}
$$

where mode conversion current is given by eq.(6).

$$
\begin{aligned}
\boldsymbol{i}^{iH} &= Y^{iHiE} v^{iE} & Y^{iHiE} &= j\eta_0 H^{iHiE} Y_{c//}^{iE} \\
\boldsymbol{i}^{iE} &= Y^{iEiH} v^{iH} & Y^{iEiH} &= (j\eta_0)^{-1} H^{iEiH} Y_{c//}^{iH}
\end{aligned}
\tag{6}
$$

From these relation of eq(2) through eq.(6), equivalent network for step discontinuity for two cases are shown in Fig.2(b)/Fig.3(b) or matrix form in Fig.2(c)/Fig.3(c). Practical mode coupling and mode conversion coefficient for two structures are given in Table II and Table III, respectively.

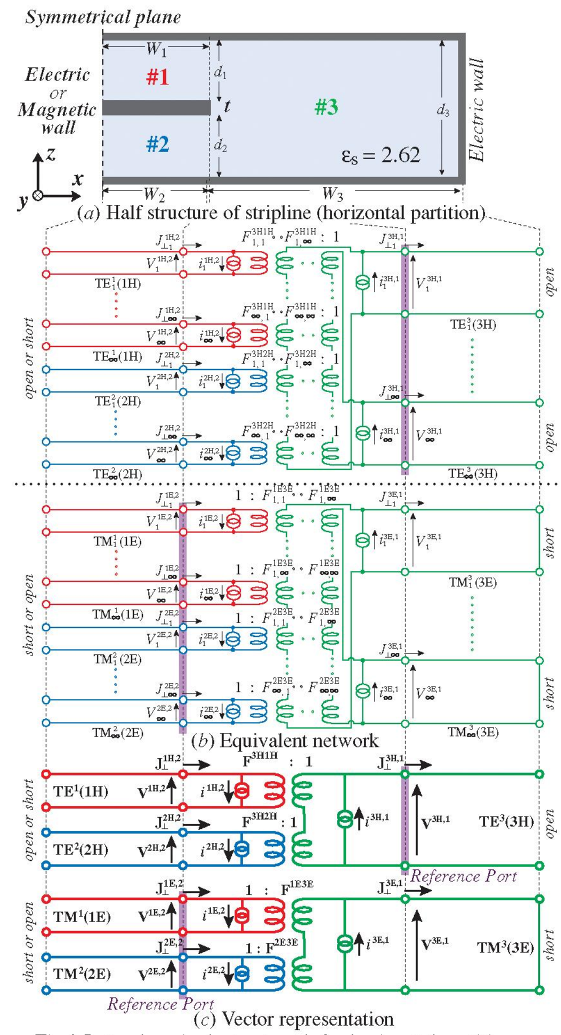


Fig.2 Lateral equivalent network for horizontal partition case

III. Formulation of Eigenmode Equation

Based on the lateral equivalent network given in Fig.2 or Fig.3, eigenvalue equations are formulated for two cases.

A. *Eigenvalue equation for horizontal partition case (Fig.2)*

When we assume that TE mode voltage at port 1 of region #3 and TM mode voltage at port 2 of region #1 and #2 in Fig.2 are unknown, eigenvalue equation (7) in Table II is formulated.

B. *Eigenvalue equation for vertical partition case (Fig.3)*

When we assume that TE mode voltage and TM mode current at port 2 of region 1 and port 1 of region 3 are unknown, eigenvalue equation (8) in Table III is formulated.

Through the formulation each immitance matrix are derived by the equivalent network given in Fig.2 or Fig.3, respectively.

IV. Calculated Results

Propagation eigenmodes of stripline are calculated by solving eigenvalue equations (7) and (8) corresponding to different partition in Fig.1, where number of height-mode is taken as parameter. Through the calculation electric wall at outside and electric/magnetic wall at center is assumed in Fig.1.

A. *Convergence with height-mode*

Propagation constant for dominant mode is always $\beta_{//} = \omega\sqrt{\varepsilon\mu_0}$ regardless height mode number. That for 2nd mode is calcu-

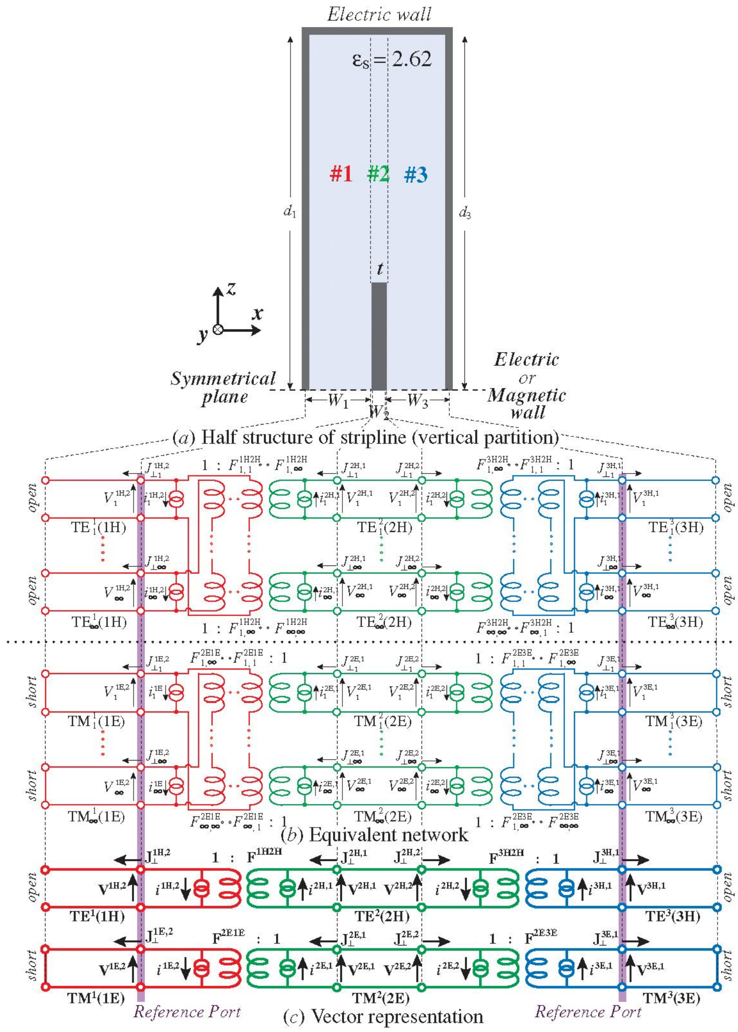


Fig.3 Lateral equivalent network for vertical partition case

TABLE II Circuit Parameters of Equivalent Network and Eigenvalue Equation for Horizontal Partition

z-dependent function (#i=1,2,3)		Mode coupling coefficient	
TE(iH) mode	TM(iE) mode	$F^{3H1H}_{m',m}=\frac{1}{d_3}\int_0^{d_1} g^{3H}_{m'}(z_3)f^{1H}_m(z_1)dz_1$	$F^{1E3E}_{n,n'}=\frac{1}{d_1}\int_0^{d_1} g^{1E}_{n}(z_1)f^{3E}_{n'}(z_3)dz_1$
$f^{iH}_m(z)=\sqrt{2}\sin\frac{m\pi}{d_i}z$	$f^{iE}_n(z)=\sqrt{\varepsilon_n}\cos\frac{n\pi}{d_i}z$	$F^{3H2H}_{m',m}=\frac{1}{d_3}\int_0^{d_2} g^{3H}_{m'}(z_3)f^{2H}_m(z_2)dz_2$	$F^{2E3E}_{n,n'}=\frac{1}{d_2}\int_0^{d_2} g^{2E}_{n}(z_2)f^{3E}_{n'}(z_3)dz_2$
$g^{iH}_m(z)=\sqrt{2}\sin\frac{m\pi}{d_i}z$	$g^{iE}_n(z)=\sqrt{\varepsilon_n}\cos\frac{n\pi}{d_i}z$	Mode conversion coefficient (#i=1,2,3)	
$h^{iH}_m(z)=\frac{m\pi}{k_0d_i}\sqrt{2}\cos\frac{m\pi}{d_i}z$	$h^{iE}_n(z)=-\frac{n\pi}{k_0d_i}\sqrt{\varepsilon_n}\sin\frac{n\pi}{d_i}z$	$H^{iEiH}_{n,m}=\frac{1}{d_i}\int_0^{d_i} g^{iE}_n(z_i)h^{iH}_m(z_i)dz_i$	$H^{iHiE}_{m,n}=\frac{1}{d_i}\int_0^{d_i} g^{iH}_m(z_i)h^{iE}_n(z_i)dz_i$

Eigenvalue equation for horizontal partition case

$$\left\{\begin{bmatrix} F^{1E3E}\vec{Y}^{3E,1}_{in}(F^{1E3E})^t & F^{1E3E}\vec{Y}^{3E,1}_{in}(F^{2E3E})^t & -F^{1E3E}Y^{3E3H}+Y^{1E1H}(F^{3H1H})^t \\ F^{2E3E}\vec{Y}^{3E,1}_{in}(F^{1E3E})^t & F^{2E3E}\vec{Y}^{3E,1}_{in}(F^{2E3E})^t & -F^{2E3E}Y^{3E3H}+Y^{2E2H}(F^{3H2H})^t \\ F^{3H1H}Y^{1H1E}-Y^{3H3E}(F^{1E3E})^t & F^{3H2H}Y^{2H2E}-Y^{3H3E}(F^{2E3E})^t & -F^{3H1H}\overleftarrow{Y}^{1H,2}_{in}(F^{3H1H})^t-F^{3H2H}\overleftarrow{Y}^{2H,2}_{in}(F^{3H2H})^t \end{bmatrix} - \begin{bmatrix} \overleftarrow{Y}^{1E,2}_{in} & 0 & 0 \\ 0 & \overleftarrow{Y}^{2E,2}_{in} & 0 \\ 0 & 0 & \vec{Y}^{3H,1}_{in} \end{bmatrix}\right\}\begin{bmatrix} V^{1E,2} \\ V^{2E,2} \\ V^{3H,1} \end{bmatrix}=0 \quad (7)$$

TABLE III Circuit Parameters of Equivalent Network and Eigenvalue Equation for Vertical Partition

z-dependent function (#i=2)		z-dependent function (#i=1,3)	
TE(2H) mode	TM(2E) mode	TE(iH) mode	TM(iE) mode
$f^{2H}_m(z)=g^{2H}_m(z)=\sqrt{2}\sin\frac{m\pi}{d_2}z$	$f^{2E}_n(z)=g^{2E}_n(z)=\sqrt{\varepsilon_n}\cos\frac{n\pi}{d_2}z$	$f^{iH}_m(z)=g^{iH}_m(z)=\sqrt{2}\sin\left(m+\frac{1}{2}\right)\frac{\pi}{d_i}z$	$f^{iE}_n(z)=g^{iE}_n(z)=\sqrt{\varepsilon_n}\cos\left(n+\frac{1}{2}\right)\frac{\pi}{d_i}z$
$h^{2H}_m(z)=\frac{m\pi}{k_0d_2}\sqrt{2}\cos\frac{m\pi}{d_2}z$	$h^{2E}_n(z)=-\frac{n\pi}{k_0d_2}\sqrt{\varepsilon_n}\sin\frac{n\pi}{d_2}z$	$h^{iH}_m(z)=\left(m+\frac{1}{2}\right)\frac{\sqrt{2}\pi}{k_0d_i}\cos\left(m+\frac{1}{2}\right)\frac{\pi}{d_i}z$	$h^{iE}_n(z)=-\left(n+\frac{1}{2}\right)\frac{\sqrt{\varepsilon_n}\pi}{k_0d_i}\sin\left(n+\frac{1}{2}\right)\frac{\pi}{d_i}z$

Mode coupling coefficient		Mode conversion coefficient (#i=1,2,3)
$F^{1H2H}_{m',m}=\frac{1}{d_1}\int_0^{d_2} g^{1H}_{m'}(z_1)f^{2H}_m(z_2)dz_2$	$F^{3H2H}_{m',m}=\frac{1}{d_3}\int_0^{d_2} g^{3H}_{m'}(z_3)f^{2H}_m(z_2)dz_2$	$H^{iEiH}_{n,m}=\frac{1}{d_i}\int_0^{d_i} g^{iE}_n(z_i)h^{iH}_m(z_i)dz_i$
$F^{2E1E}_{n,n'}=\frac{1}{d_2}\int_0^{d_2} g^{2E}_{n}(z_2)f^{1E}_{n'}(z_1)dz_2$	$F^{2E3E}_{n,n'}=\frac{1}{d_2}\int_0^{d_2} g^{2E}_{n}(z_2)f^{3E}_{n'}(z_3)dz_2$	$H^{iHiE}_{m,n}=\frac{1}{d_i}\int_0^{d_i} g^{iH}_m(z_i)h^{iE}_n(z_i)dz_i$

Eigenvalue equation for vertical partition case

$$\left\{\begin{bmatrix} -F^{1H2H} & Y^{1,2} & 0 & 0 \\ 0 & (F^{2E1E})^t & 0 & 0 \\ 0 & 0 & -F^{3H2H} & -Y^{3,2} \\ 0 & 0 & 0 & (F^{2E3E})^t \end{bmatrix}\begin{bmatrix} Y^{2H}_{11} & 0 & -Y^{2H}_{13} & 0 \\ 0 & Z^{2E}_{22} & 0 & Z^{2E}_{24} \\ -Y^{2H}_{31} & 0 & Y^{2H}_{33} & 0 \\ 0 & Z^{2E}_{42} & 0 & Z^{2E}_{44} \end{bmatrix}\begin{bmatrix} (F^{1H2H})^t & 0 & 0 & 0 \\ -Y^{2,1} & -F^{2E1E} & 0 & 0 \\ 0 & 0 & (F^{3H2H})^t & 0 \\ 0 & 0 & -Y^{2,3} & -F^{2E3E} \end{bmatrix} - \begin{bmatrix} \overleftarrow{Y}^{1H,2}_{in} & 0 & 0 & 0 \\ 0 & \overleftarrow{Z}^{1E,2}_{in} & 0 & 0 \\ 0 & 0 & \vec{Y}^{3H,1}_{in} & 0 \\ 0 & 0 & 0 & \vec{Z}^{3E,1}_{in} \end{bmatrix}\right\}\begin{bmatrix} V^{1H,2} \\ J^{1E,2} \\ V^{3H,1} \\ J^{3E,1} \end{bmatrix}=0 \quad (8)$$

lated and shown in Fig.4 as a function of considered height-mode in region 3 for case 1 and Region 1 for case 2. Propagation constant always converges to a certain value with height mode for two cases. They converge to the same value regardless of different equivalent network parameters of two cases.

B. Calculation of propagation constant

Frequency characteristics of propagation constants for mode up to 10th are calculated and shown in Fig.5. Agreement of the calculated results by two ways is good.

C. Calculation of field distribution of lower mode

Field distribution of the dominant TEM mode and 2nd mode are calculated by two ways and shown in Fig.6. It is interesting but is matter of course that calculated field distribution by two ways are the same.

V. CONCLUSION

Propagation eigenmodes for stripline are calculated by two different partition but similar lateral equivalent network. How to derive the equivalent network for partition of stripline along horizontal and vertical direction is explained. Two equivalent networks are applied to the analysis of practical stripline. Agreement of calculated results by two ways can demonstrate the usefulness of the present lateral equivalent network.

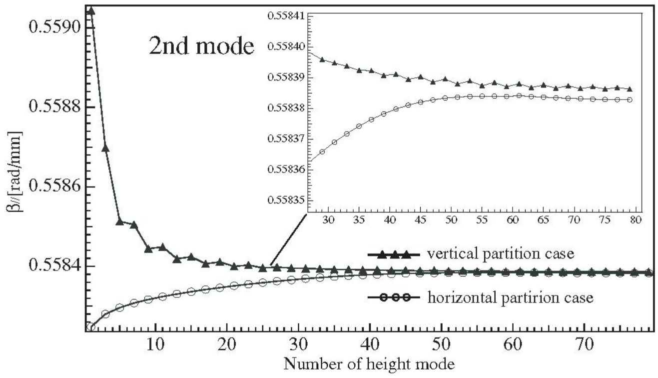


Fig.4 Convergence of propagation constant with height-mode

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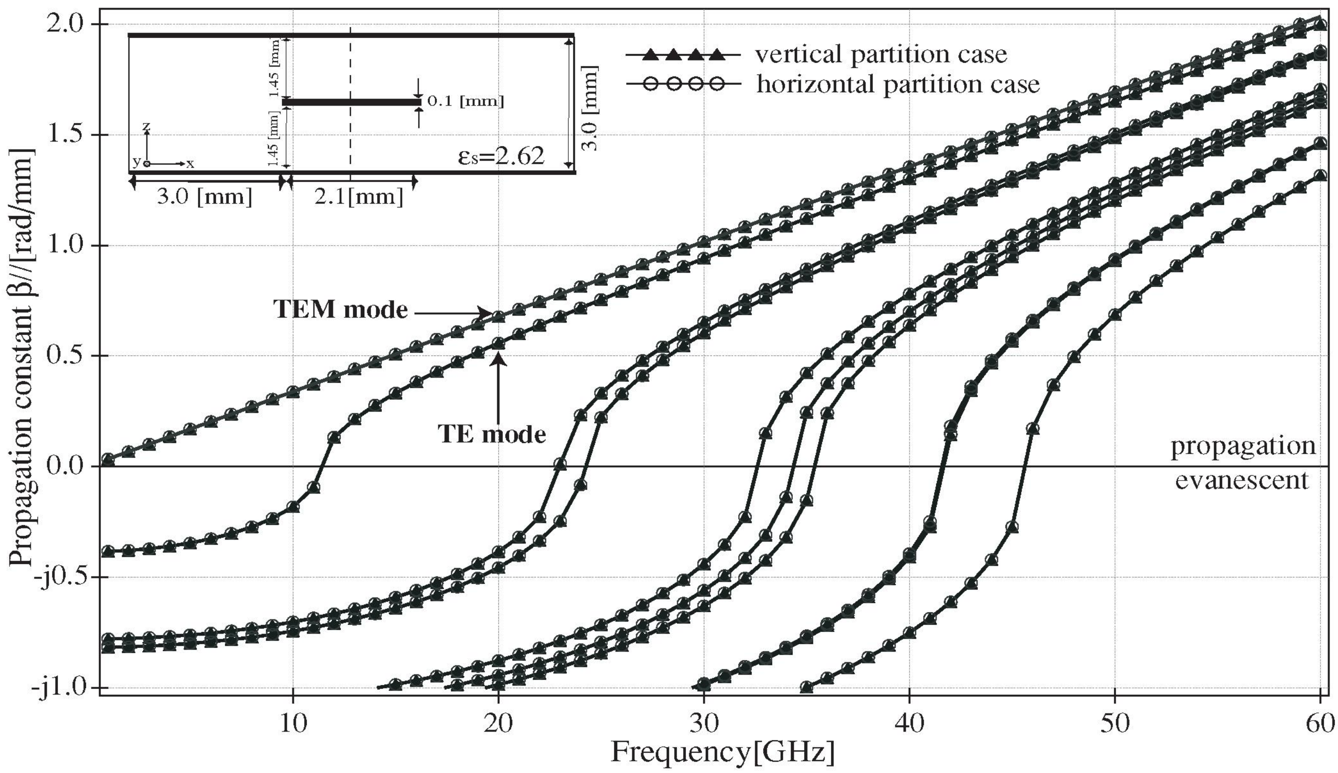

Fig.5 Frequency characteristic of propagation constant calculated by two ways

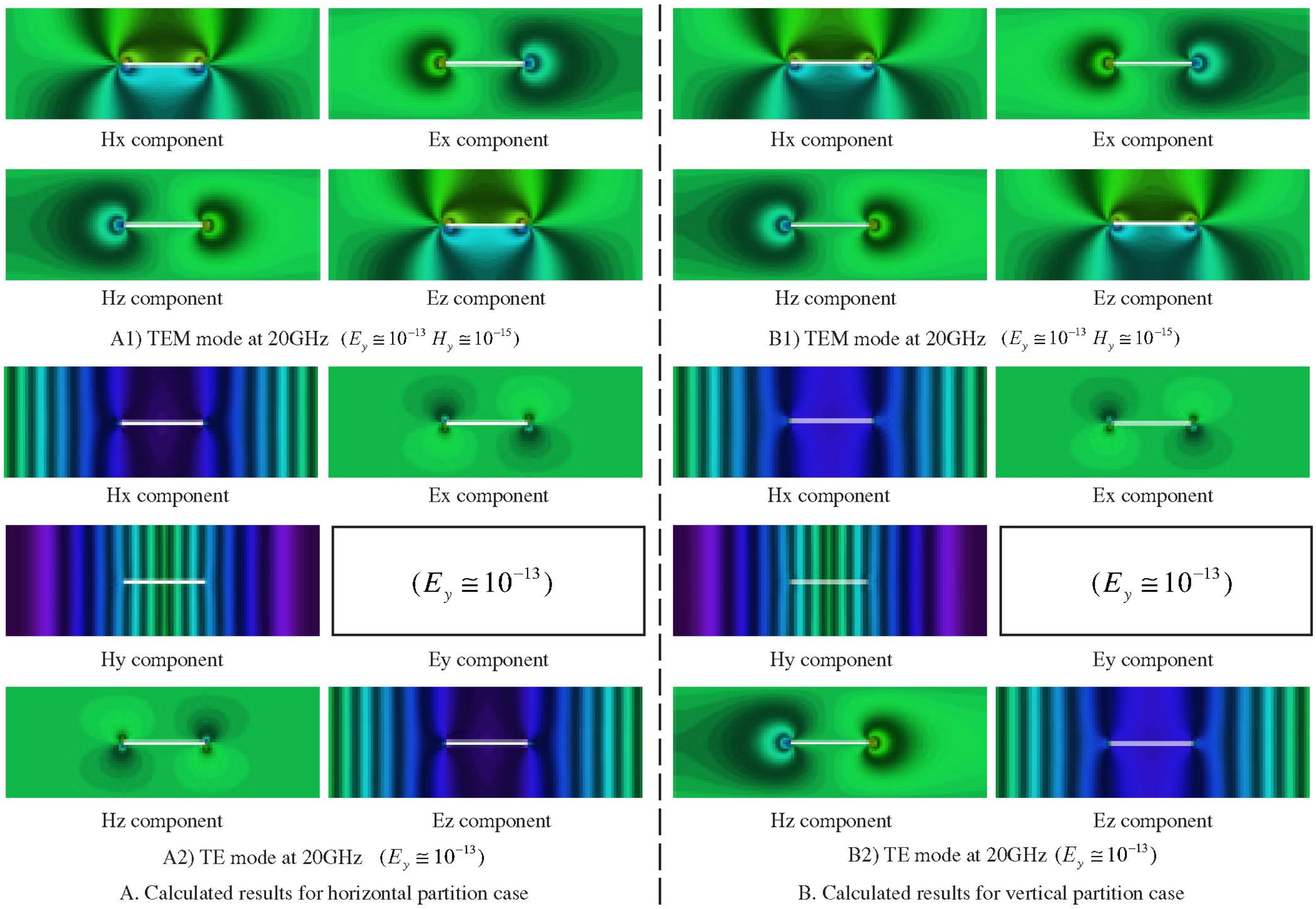

Fig.6 Field distribution of the dominant TEM mode and 2nd mode of TE calculated by two ways at 20GHz