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Allocation Mechanism in Economic Organization

——A Synthesis——

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Abstract: In this paper, some resource allocation mechanism are reconstructed in uniformed framework to compare with those economic performance. Especially, this paper focalizes the informational aspects of those mechanism, but the aspects of incentive compatibility is disregarded.

1. INTRODUCTION

The study on resource allocation process has been the one of main subjects in Micro Economic theory. For the past thirty years, many papers on allocation mechanism have been published and some of them have already been a basic theme in economic thought. Arrow and Hurwicz [4] which draws a long series of articles and gives a historical overview is the one of the most important publications in this field. This book tells us that the stream of investigation of resource allocation mechanism had been changed at the end of 1970. In an earlier time, the motive to study an allocation process had been based on a viewpoint of the feasibility and optimality of economic planning. After the end of 1970, the learning work on allocation mechanism has been taken up from a viewpoint of the possibility of design of incentive compatible mechanism and of the theory of social choice. Therefore, the related recent works which are based on

game theoretic approach have been given a generic name so-called "Mechanism Design". To our regret, we have to leave our works to review this recent studies, because its direction of development is vague for the present.

In this paper, we introduce a general framework in which some resource allocation mechanism can be uniformly described and made a comparison between these economic performance. This is the modified of the framework which was pioneered by Hurwicz [11] [12], and developed by Aoki [1] and others. Section 2 offers an introductory discussion on an allocation problem. From a viewpoint of economic planning, we clarify the issues in an allocation problem.

Section 3 introduces the general framework. In section 4, employing this general framework, we reconstitute some allocation mechanisms which were investigated by Arrow-Hurwicz [3], Malinvaud [15], and Heal [6]. Section 5 presents some discussions on criterions by which economic performance of each mechanism is passed judgment. Last section mentions some concluding remarks.

2. INTRODUCTORY DISCUSSIONS

This section offers a short discussion on the problem incidental to the consistency of decision making in large economic organization. So, we shall suppose the central planning economy in which every economic adjustment processes are controlled by special economic agent called "the Central Planning Board (C. P. B)". For the present, We assume that C. P. B has a full knowledge of an economic environment to be formed of various economic agents, their preferences and production technologies, available resources, and so on. Then, provided that C. P. B can hold a perfect information of the final goal of an economy that is just an eco-

nomic environment, the optimal decision that C. P. B makes a choice under some criterions is to find a most efficient resource allocation of consumption and production "x" which possibly attains the final goal of an economy. Mathematically, it is a constraint maximization problem as follows,

Find x such that Max $U(x)$ *subject to* $x \in D$

where $U(x)$ is the objective function of C. P. B, for instance, the index of the social welfare level, and x represents a resource allocation vector which is an element of the set D. Needless to say, D is the set of feasible resource allocation.

In this place, we would like to point out tow issues in an economic planning procedure formulated in the above. The first is as follows. Even if C. P. B can hold a full knowledge of the set D which is induced from the production technology and the initial resource allocation, and furthermore if he can have a perfect information on$U(x)$ which may be constructed from each individual preference in an economy, he may not be able to find an efficient allocation vector x. Because, for C. P. B, to solve this maximization problem must need an enormous capacity of calculation. (In detail, see Hayek [5].) The second concerns the ability of C. P. B to gather various information. It seems to be an unrealistic assumption that C. P. B perfectly knows each preference and production technology of all economic agents. The supposition that each agent merely knows his own prefence and technology is more natural. That is to say, to gather perfect information wiht regard to an economic environment needs enormous cost and time. So, the information gathering ability of C. P. B must necessarily become to be limited.

These contexts mentioned above have been become clear and clear, since so-called "the socialism controversy" which might be inspired by

Osker Lange had been began in the 1930'. As is well known, the fact that the "centralistic" socialism as an economy which C. P. B must control all economic adjustment processes never be well-workable is broadly recognized as results in this controversy.

Presupposing that even the C. P. B of ability can't gather full information and accurately analysis it, to assume that each economic agent knows only his own characteristics is more plausible. In the line of an above thought, Hurwicz [10], [12] and Malinvaud [15] made a proposal to reconstruct the studies on resource allocation process as a process under "informationally decentralized systems". The term of "informationally decentralized systems", roughly speaking, represents an economic systems in which each individual has a full knowledge of his own characteristics and merely has an imperfect informion on the other. Thereafter, in the stream of same critical mind, many papers on the possibility of economic planning for the informationally decentralized economy have been published. In the following sections, some of those studies will be reconstructed under an uniformed framework.

3. THE GENERAL FRAMEWORK

We shall begin our works to define some concepts on the resource allocation process. These concepts are based on the consideration that a behavior of rational agent must be explicitly analyzed as a part of working of the economic organization. That is to say, it means that the economic behavior of each agent must be regarded as a basic part of the concerned economy. Furthermore, to give an uniform description to various types of the allocation process, these concepts require to be defined with respect to various economic functions which are production, consumption, initial holding of resource, and so on.

Firstly, we shall define the concept of "economic organization".

Economic Organization: The economic organization is composed of two parts which are an economic environment and an adjustment process.

Definition 1: The pair (e, θ) of an economic environment and an adjustment process is called the "Economic Organization".
The economic environment is denoted by e and θ represents the adjustment process.

Definition 2: The economic environment e is a set whose components are members of an economic organization and various economic information which are decentralizlly held by each member.

Definition 3: The each member of an economic organization is an economic agent who makes a decision by himself, and they are classified as follows with respect to their economic functional.

Def 3-1: Helmsman: The helmsman is an agent who holds a full knowledge on a final target of the "Economic Organization". The set of helmsman is denoted by H.

For instance, if the helmsman will be a consumer or household, then he perfectly knows his utility function and his initial holding of resource. If he will be the C. P. B, then he can get a perfect information on a final goal of the concerned national economy.

Def 3-2: Custodian: The custodian is an agent who knows an initial allocation of available resources and manages its utilization. The capital C represents the set of custodian.

Def 3-3: Producer: The producer is an agent who completely knows the production process or the production function and executes the production of goods or services. P represents the set of producer.[(1)]

The economic information held by each agent are specialized as follows.

G ; Commodity space.

W_i ; an initial endowment of i-th custodian.

Y_j ; a production possibility set of j-th producer. Of course, $Y_j \subseteq G$.

H_k ; a preference relation of k-th helmsman defined on Z_k.

Z_k ; an admissible demand set of k-th helmsman. Of course, $Z_k \subseteq G$.

Where three suffixes i, j, and k imply a number of each agent. These information are held by each agent at initial time of adjustment process and will be called the "primitive information". Moreover, G is a subset of n -dimensional real "Euclidean Space". Of course, n is a number of commodity.

Now, we can denote the economic environment e as follow by using these notations,

$$e = (N, G, W, Y, (H_k, Z_k)) \quad k = 1, 2, \cdots\cdots$$

where $N = H \cup C \cup P$, $w = (w_1, w_2 \cdots w_n \cdots)$, and Y is the "Cartesian product" of Y_j. Obviously, H_k is defined on Z_k. Therefore, we write them as the pair (H_k, Z_k) .

Here, we shall introduce a new agent called "messenger" to define the adjustment process θ.

Def 3-4: Messenger: The messenger is an agent who has a role to transmit various information to each agent.

We assume that every economic information diffuses among each agent through the messenger. The adjustment prucess θ is composed of two parts. The one is a process of exchanging information among agents, the other is a decision rule which orders to stop the information exchanging process and to shift to real economic actions. We will call the rule controlling the process a "Dialog rule" and the rule stopping the process a "Decision rule". These tow rules are constructed as follows.

Dialog rule:

M_o; M_o is the set of information which is transmitted to each agent

by the messenger.

M_I; M_I is the set of information which each agent transmits to the messenger.

f_i; f_i represents a reaction of i-th agent to information transmitted by the messenger.

η; η represents a response of the messenger to information transmitted by each agent.

where f_i is called the reaction rule and η is the control rule in economic planning theory. Moreover, for instance, if the information transmitted by the messenger is market prices of goods, then M_o is a subset of positive orthant of n-dimensional real "Euclidean Space".

The information exchanging process starts at time when the messenger transmits some type of resource allocation m_o to each agent such that $m_o \in M_o$. Then, each agent reacts to $m_o$and send a reply m_i derived from $f_i$to the messenger. Of course, $m_I \subseteq M_I$. After that, the process will continue according to Dialog rule.[2]

Decision rule:

T; T is a rule which determines a time to stop the information exchanging process.

A; A is a rule that the result yielded by the information exchanging process corresponds to "real" economic actions.

Under the above notations, the adjustment process θ can be described as follow.[3]

$$\theta = (M_o, M_I, f, \eta, T, A)$$

where f is of the form such that $f = (f_1, f_2, \cdots, f_i, \cdots, f_j, \cdots)$.

In the next section, we shall reconstruct some resource allocation mechanisms under the framework defined as above.

4. SOME ALLOCATION MECHANISMS

In this section, we shall describe some resource allocation mechanisms. These mechanisms are severally called "Lange-Allow-Hurwicz process", "Malinvaund Process", and "Non-price Planning Procedure" by Heal [8].

4-1: Lange-Allow-Hurwicz Process

The Lange-Allow-Hurwicz Process is essentially equivalent to a resource allocation mechanism which is broadly recognized as the neo-classical market mechanism or the "tâtonment process" in the Walrasian sense. This process can be summarily reconstructed in our framework as follows.

:e: The helmsman and the custodian are the same and unique agent who is called a "representative household". The producer is a "firm", and the messenger is a so-called "Auctioneer".
And, the G is a subset of the positive orthant of n-dimensional real "Euclidian space". The w_i is a subset of G, and Y_j is a convex subset of G. The H_k is a "complete ordering" defined on Z_k which is a subset of G. (Note that the suffixes i and k in above mean an only agent, because the representative household is an unique agent.)

:θ: The θ is summed up by following form. This adjustment process is totally a same sort of the "Walrasian tâtonment process".

:M_o: The M_o is the set of price vectors.

:M_I: The M_I is the set of excess demand vectors of goods.

:f: The f_i and f_j are just reactions of a "representative household i" and each "firm j" which follow in below procedure.

[f_i] ; The representative household makes a choice z_i^* which is a

highest rank w. r. t his preference H_h subject to the following constraint:

$$m_o z_i^* < m_o w_i + I_i \ , \ m_o \in M_o \cdots [1]$$

where I_i is a positive income-transfer of which the existence is supported by the strong convexity of production possibility set $Y_h^{(4)}$. (Note that the representative household is an only agent. Therefore, the suffix i in the above formula [1] means a special "i".) And, he transmits a m_i to "Auctioneer" such that

$$m_i = z_i^* - w_i \ , \ m_i \in M_I.$$

$[f_j]$; The firm make a choice y_j^* which satisfies

$$Max\ m_o y_j = m_o y_j^* \qquad for\ all\ y_j \in Y_j.$$

And, he transmits a m_I^j to "Auctioneer" such that

$$m_I^j = -y_j^*.$$

:Π: The "Auctioneer" revises his initial information m_o according to the procedure described by the following differential equations:

$$\dot{m}_o^r = \begin{cases} O, \ when\ m_o^r = O\ and\ \Sigma m_i^r < O. \\ \qquad\qquad\qquad\qquad\qquad for\ all\ r. \\ F_r\ (\Sigma m_i^r), \ othewise. \end{cases}$$

where F_r ($\cdot$) is a sign-preserving function of Σm_i^r, *with* F_r $(O) = O$, and differentiable with $F_r' > \overset{(5)}{O}$. Of course, the suffix r represents a number of goods, and $m_o = (m_o^1, \cdots, m_o^r, \cdots)$ is a price vector. The $\dot{m}_o^r$ represents a time differential of m_o^r such that dm_o^r/dt.

And, he transmits a revised information to the representative household and each firm.

:T: The dialog is stopped at a time when $\dot{m}_o^r = O$, for all r.

:A: The action of each agent is decided with according to the information held by him at the time when $\dot{m}_o^r = O$.

4-2: Malinvaud Process

Malinvaud [15] showed a possibility of efficient resource allocation process which would be controlled by Central Planning Board (C. P. B) under some conditions. That is, from the viewpoint of economic planning, it showed the feasibility of an efficient resource allocation in centralized economy and discussed how the problem that C. P. B found an efficient allocation could be solved.

:*e*: The helmsman and the custodian are the same and unique agent who is called a "Central Planning Board (C. P. B)". The producer is a "firm", and the role of a messenger is fulfilled by C. P. B. And, the G is a subset of the positive orthant of n-dimensional real "Euclidian space". The $w_i$is a subset of G, and Y_j is a convex subset of G. The H_k is a "complete ordering" defined on Z_k which is a subset of G. (*Note* that the w_i and H_k severally mean an initial holding and a preference of an uniqe agent C. P. B.)

:θ: The adjustment process θ is summed up by following form.

:M_o: The M_o is the set of price vectors.

:M_I: The M_I is the set of demand vectors of commodities.

:*f*: Under the condition that the information m_o from C. P. B is exogenously given, the firm i finds a m_I^{i*} such that

$$Max\ m_o\, m_I^i = m_o\, m_I^{i*} \qquad for\ all\ m_I^i \in Y_i$$

And, he transmits a m_I^{i*} to C. P. B.

:η: C. P. B revises his initial information m_o according to below procedure.

[step 1] ; C. P. B finds an allocation z_k^* which is a highest rank w. r.t his preference H_k subject to the following conditions such that

$$z_k^* = \Sigma m_l^i + w > O \qquad for\ all\ m_l^i \in \hat{Y}_i$$

where $\hat{Y}_i = \Sigma \lambda^j m_l^{i*}$ *and* $\Sigma \lambda^j = 1$.

[step 2] ; Secondly, he finds a price vector m_o^* which satisfies:

$$m_o^* \overline{z_k} > m_o^* z_k, \text{ and}$$

$$m_o^* m_l^{i*} > m_o^* m_l^i \qquad for\ all\ m_l^i \in \hat{Y}_i$$

where $\overline{z_k}$ is every higher rank of an allocation than z_k^* w. r. t his preference H_k.

And, he transmits a m_o^* to each firm.

:T: The dialog is stopped at a time t which is exogenously given.

:A: The action of each agent is decided with according to the information held by him at the time t.

The characteristics of Malinvaud Process is the price exchanging mechanism controlled by C. P. B. Gathering information through the dialog, C. P. B constructs a new set $\hat{Y}_i$ of allocation vectors. That is to say, C. P. B approximates $\hat{Y}_i$ to the production feasible set of each firm, and he find a new correspondence code m_o^*, solving the problems of [step 1] and [step 2] by using $\hat{Y}_i$.

4-3: Non-price Planning Procedure

Heal [6] showed a resource allocation mechanism which C. P. B would control by means of non-price code. It showed that C. P. B could find an efficient allocation by using a marginal productivity of resource in each firm. This allocation mechanism is called "Heal's Process" in economic planning theory, and is reconstructed in our framework as follows.

:e: The helmsman and the custodian are the same and unique agent who is called a "Central Planning Board (C. P. B)". The producer is a "firm", and the role of a messenger is fulfilled by C. P. B. And, the G is a subset of the positive orthant of n-

dimensional real "Euclidian space". The w_i is a subset of G, and Y_j is a subset of G. (Note that the Y_j may be a non-convex subset of G.) *The* H_k is a "complete ordering" defined on Z_k which is a subset of G (Note that the w_i and H_k severally mean an initial holding and a preference of an uniqe agent C. P. B.)

:θ: The adjustment process θ is summed up by following form.

:M_o: The M_o is the set of input-allocation vectors of each firm.

:M_I: The M_I is the set of vectors whose components represent a marginal productivity of the input-allocation of resources at each fim.

:f: Under the information m_o from C. P. B which appoints the menu of input resource for each firm, the firm i calculates the output level and the marginal productivity of input resource.

And, he transmits the marginal productivity vector $m_I{}^i$ to C. P. B.

:η: C. P. B revises his initial information m_o according to the procedure described by bellow differential equations:

$$\dot{m}_o = \begin{cases} m_I{}^i - \dfrac{1}{|K_j|}\sum_j m_I{}^i & \text{for all } i \in K_j \\ O & \text{for all } i \notin K_j \end{cases}$$

where $|K_j|$ represents a number of firm belong to K_j. And, the set K_j is the set of firms in which the marginal productivities of input resources are higher than an average of those over all firms.[6] Of coruse, $\dot{m}_o$ represents dm_o/dt.

And, C. P. B transmits the reviced information to each firm.

:T: The dialog is stopped at a time t which is determined by C. P. B.

:A: The action of each agent is decided with according to the information held by him at the time t.

The characteristics of Heal's Process is the correspondence of C. P. B to the information of each firm. That is to say, it is a procedure that C. P. B allocates more of input to firm where its marginal productivity is higher than the average over all firms. In this sence, it is called "non-price planning procedure". One side, in both of the previous processes the centre uses price vector as information, therefore these are called "price-guided planning procedure" by Heal [8] .

5. SOME CRITERIA OF ASSESSMENT OF ALLOCATION PROCESS

This section offers a short discussion on some criteria of assessment of allocation process. Aoki [1] required three standards as criteria by which allocation processes are assessed. These standards are,

(1) that each process must have a characteristic that the process monotonically converges to an efficient resource allocation ;

(2) that the cost derived from exchanging information in each process should be lowered ;

(3) that the dialog rule in each process must be incentive-compatible with all agents.

The first two standards imply that any allocation process must be required to have the strict monotonicity property that if each step in allocation process leads to a new allocation with a higher value of objective function of Helmsman than that given by the previous allocation. Further more, these imply that an allocation attained by any process can be feasible w.r.t the availability of resources even if the dialog process was stopped at any process time t .

In these respect, Malinvaud [15] discussed the monotonicity and feasibility criteria as standards of points [1] and [2] in above. The monotonicity

criterion means what any process should monotonically converge to an efficient allocation. That is, it means that an allocation assigned at the process time t must be a higher rank w.r.t the preference of Helmsman than that assigned at any previous process time.

The feasibility criterion implies that an allocation assigned at any process time t must be feasible w.r.t the constraint of available resources. The these two criteria are clearly satisfied by *Malinvaud Process* and *Non-price Planning Procedure*, but not by *Lange-Allow-Hurwicz Process*. And, in respect of the standard [2], Mount and Reiter [16] discussed that the cost and volume of exchanging information should be measured by means of the dimension of commodity space. It means that the cost of gathering information corresponds to the number of goods which may allocate in each process.

The standard [3] is a most important criterion to evaluate an economic performance of each allocation process. As this standard, Aoki [1] required that the dialog rule of each process should have a property to eliminate a "free-raider" from the process and the decision rule should be fulfilled by each agent under less compulsion.

Note that these standards are exactly criteria by which each adjustment process of resource allocation is assessed. However, each process has different characteristics when it works in an other economic environment. For instance, *Non-price Planning Procedure* has the feasibility and monotonicity properties when it works in convex and nonconvex environments of Y, but *Malinvaud Process* and *Lange-Allow-Hurwicz Process* do not have these proprties any longer in a nonconvex environment of Y. It means that the assessment criterion of an economic organization must be satandards by which the pair (e, θ) of an economic environment and an adjustment process is evaluated.

6. SOME CONCLUDING REMARKS

In above sections,we construct the general framework to present an uniform description for some different allocation processes.The framework like this may be more suitable for the description of tâtonment type of an adjustment process.Above three processes can be called the "tâtonment type",because they have a common characteristics such that the C.P. B or the auctioneer who are an only agent in each economic organization will search an efficient allocation of resources by means of information independently transmitted from each agent.That is,in the process of the *Dialog rule* of an econmy that the "Helmsman" is an only agent,each other agent only sends a information M_I according to his own algorithm.

The remained work is that the framework should be more expanded to present general description for the incentive compatibility of resource allocation processes.Because the incentive compatibility plays an important role for the stability of an economic organization,we need have the framework to pursue an argument to its logical conclusion on the incentive compatibility.

Footnotes

(1) These terms,"Helmsman","Custodian" and "Producer" were introduced by Koopmans and Montias [13],and were redefined by Aoki [1].

(2) Hurwicz [12] and Malinvaud [15] showed that the dialog rule in the allocation process concerned should satisfy some conditions when it would start at initial process time.

(3) These notations and the difinition θ of the adjustment process were basically introduced by Aoki [1].However,these are drastically refined here, and the *Decision rule* T is originally revised in this paper.

(4) At the equilibrium point, the I_i is equal to $\Sigma\, m_o\, y_j^*$. Still more, in the condition of the strong convexity of y_j^*, the I_i $(= \Sigma\, m_o\, y_j^*)$ has a positive value. In detail, see the chapter 3 of Arrow and Hahn [2].

(5) On the sign-preserving function $F_r(\cdot)$, see the chapter 11 of Arrow and Hahn [2].

(6) In this paper, Heal's process is more simply described in our framework. On the original formulation of it, see Heal [6] and [7].

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